ARIZONA ACADEMIC CONTENT STANDARDS

MATHEMATICS

Approved by Arizona State Board of Education
June 24, 2008

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Mathematics Standard Articulated by Grade Level

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STATE REPRESENTATION

Teachers who worked on the revision and articulation of the Mathematics Standard represented the schools, districts, and organizations listed below. The goal was to have representation from large and small districts, urban and rural schools, and geographic and ethnic diversity.

American Christian Academy
Amphitheater School District
Arizona School for the Arts
Arizona State University
Avondale Elementary School District
Blue Ridge Unified School District
Cartwright Elementary School District
Cave Creek Unified School District
Chandler Unified School District
Creighton School District
CRESMET (ASU)
Deer Valley Unified School District
Douglas Unified School District
Flowing Wells Unified School District
Gilbert Public Schools
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Indian Oasis-Baboquivari Unified School District
Kyrene School District
Littleton Elementary School District
Madison School District
Marana Unified School District
Mesa Public Schools
Nogales Unified School District
Northern Arizona University
Osborn School District
Paradise Education Center
Paradise Valley Unified School District
Peoria Unified School District
Phoenix Union High School District
Pinal County Educational Service Agency
Rio Salado Community College
Rodel Foundation
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Salt River Pima-Maricopa Community Schools
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Tempe Union High School District
Tucson Unified School District
University of Arizona
Vail School District
Washington Elementary School District
Wickenburg Unified School District
Yavapai Community College
Yuma School District

EXTERNAL REVIEWERS/CONSULTANTS

Jinfa Cai, Ph.D., is a professor of the Department of Mathematical Sciences and the School of Education at the University of Delaware. His research interest is related to how students learn mathematics and solve problems, and how teachers can provide and create learning environments so that students can make sense of mathematics. He seeks answers to these questions in various educational contexts, both within and across nations. He has published nearly 100 papers in national and international peer-reviewed journals and books. Currently, he is directing a 5-year longitudinal study of curricular effect on students' learning, funded by the National Science Foundation.

Valerie DeBellis, **Ed.D., (Rutgers University)** works as a Mathematics Education Consultant with Discrete Teaching and is a co-author of educational materials that focus on the learning of discrete mathematics; among them is the two-volume series, *Navigating Through Discrete Mathematics*, published by NCTM. Her research interests include understanding how affect and cognition interact in the context of mathematical problem solving.

Sarah Sword, Ph.D., (Michigan State University) is a Research Scientist at Education Development Center. A student of Christel Rotthaus, she received her PhD in Commutative Algebra from Michigan State University. At EDC, Sarah directs a program supporting schools as they implement EDC's new mathematics curricula, Think Math! (available from Houghton Mifflin Harcourt) and CME Project (available from Pearson). She also directs Center for the Scholarship of School Mathematics, which is currently offering a program for university faculty who teach doctoral students in mathematics education.

John Woodward, **Ph.D.** is a distinguished professor in the School of Education at the University of Puget Sound in Tacoma, Washington. The majority of his research since 1989 has focused on mathematics education and technology-based instruction. One of his most recent projects was a collaborative, five-year program of research that examined methods for helping students with disabilities succeed in standards-based mathematics instruction in grades 4 through 8. He has co-authored four technology-based instructional programs, and is the senior author of *Transitional Mathematics* and *Fact Fluency and More!* He has published over 80 articles in professional education journals. His work is cited in considerable detail in the recent Instructional Practices Report from the National Mathematics Advisory Panel.

The Arizona Department of Education (ADE) acknowledges the contributions of the Washington Office of Superintendent of Public Instruction in allowing the ADE to incorporate examples and wording from their Washington State K-12 Mathematics Standards.

Mathematics Standard Articulated by Grade Level

INTRODUCTION

The Arizona Mathematics Standard Articulated by Grade Level describes a connected body of mathematical understandings and competencies that provide a foundation for all students. This standard is coherent, focused on important mathematics, and well articulated across the grades. Concepts and skills that are critical to the understanding of important processes and relationships are emphasized.

The need to understand and use a variety of mathematical strategies in multiple contextual situations has never been greater. Utilization of mathematics continues to increase in all aspects of everyday life, as a part of cultural heritage, in the workplace, and in scientific and technical communities. Today's changing world will offer enhanced opportunities and options for those who thoroughly understand mathematics.

Communication, problem solving, reasoning and proof, connections, and representation are the process standards as described in the *Principles and Standards for School Mathematics* from the National Council of Teachers of Mathematics (NCTM). These process standards are interwoven within each of the content strands of the Arizona Mathematics Standard and are explicitly connected to the teaching of specific performance objectives in the grade level documents. The process standards emphasize ways to acquire and apply the content knowledge.

Mathematics education should enable students to fulfill personal ambitions and career goals in an informational age. In the NCTM *Principles and Standards* document it asks us to "*Imagine a classroom, a school, or a school district where all students have access to high-quality, engaging mathematics instruction. There are ambitious expectations for all, with accommodations for those who need it".*[1] The Arizona Mathematics Standard Articulated by Grade Level is intended to facilitate this vision.

BACKGROUND

The State Board of Education adopted the Mathematics Standard Articulated by Grade Level in 2003 to define what Arizona students need to know and be able to do at each grade level through the end of tenth grade. Developed by a committee comprised of a diverse group of educators, this standard was written in response to the requirements of *No Child Left Behind Act of 2001* (NCLB).

RATIONALE

In 2007 the State Board of Education began the process for increasing the high school graduation requirement in mathematics from two to four years. This requirement was

[1] National Council of Teachers of Mathematics, Principles and Standards for School Mathematics, NCTM Publications, Reston, VA, 2000, p. 3.

approved in December 2007 effective with the graduating class of 2013. This increase, along with the need to complete a periodic review of the standard, prompted the Arizona Department of Education to initiate the process of refining and rearticulating the Mathematics Standard. This refinement and articulation project began in June 2007 and was completed in June 2008.

METHODOLOGY

Work teams representing populations from around the state were formed. These groupings were comprised of large and small schools, rural and urban schools, and were ethnically diverse. Included were classroom teachers, curriculum directors, mathematics teacher leaders, Career and Technical Education teachers, second-career teachers, and university/community college faculty. The goal was to revise and articulate the Mathematics Standard K-12 to align with the increased state requirement of four years of high school mathematics.

The mathematics revision teams utilized the National Council of Teachers of Mathematics *Principles and Standards* as a reference in the development of the revised Mathematics Standard. Additionally, the findings and recommendations from the National Mathematics Advisory Panel, the American Diploma Project Benchmarks, the National Assessment of Educational Progress Framework, the Curriculum Focal Points, the Framework for 21st Century Skills, and other states' frameworks were used as guiding documents.

The revision grade level teams created draft documents with performance objectives articulated to the appropriate grade levels. Over a period of months, these teams and smaller sub-committees of teams refined the draft documents based on clarity, cohesiveness, and comprehensiveness. Reasonableness, usefulness, and appropriateness were key guidelines for the articulation process. The measurability of each performance objective was also a consideration.

External reviews by nationally recognized consultants brought a broader perspective to the refinement process. Another important step in the process was the gathering of public comment. In March 2008, drafts of the Revised Mathematics Standard Articulated by Grade Level, along with a survey to gather feedback, were posted on the Arizona Department of Education website. This provided the public with easy access to the documents, and a survey allowed reviewers a means for submitting comments. Also, crosswalks were created from the Draft 2008 Mathematics Standard to the 2003 Mathematics Standard and were posted on the website. The public had the opportunity to submit comments and suggestions, either electronically or in writing, until the survey closing date of March 28, 2008. Additionally, five public hearings were held in March throughout the state offering further opportunities for public feedback.

After all the public comments were collected, organized, and categorized by grade level and topic, the revision teams met to determine what modifications to the standard document would be appropriate. Upon completion of the revision work, crosswalks were created to assist educators with the transition

from the 2003 Arizona Mathematics Standard Articulated by Grade Level to the revised 2008 Mathematics Standard.

ORGANIZATION OF THE MATHEMATICS STANDARD

The Mathematics Standard Articulated by Grade Level is divided into five main strands:

- Number and Operations
- Data Analysis, Probability, and Discrete Mathematics
- Patterns, Algebra, and Functions
- Geometry and Measurement
- Structure and Logic.

Each strand is divided into concepts that broadly define the skills and knowledge that students are expected to know and be able to do. Under each concept are performance objectives (POs) that more specifically delineate the ideas to be taught and learned.

The comprehensive document (K-12) is designed so that teachers can read the performance objectives across grade levels to incorporate learning from previous, current, and future grade levels. The standard is separated into two separate documents due to the addition of College Work Readiness (grades 11-12). The first document spans grade levels K through 6, and the second document covers grades 7 through College Work Readiness. Viewing the Mathematics Standard document from left to right helps the teacher to see the mathematics continuum across the grade levels. There is a purposeful clustering of performance objectives in order to emphasize certain key understandings. Every effort was made to eliminate repetitions. The intent was to build on the learning in previous grade levels, connect important ideas, and highlight new content each year. This coherency supports students in developing new understandings and skills. Looking down each individual column enables a teacher to see the performance objectives that students are expected to know and be able to do at any grade level.

This organization does not imply that the teaching and learning of mathematics should be fragmented or compartmentalized. Mathematics is a highly interconnected discipline; important mathematical ideas from all five mathematics strands need to be continuously integrated as needed to make meaning and connections to other concepts and performance objectives. In each grade level document, these connections are highlighted.

The order of the strands, concepts, and performance objectives (POs) in the Mathematics Standard document are not intended to be a checklist for mathematics instruction. Mathematical concepts develop with a spiraling of ideas/skills that are interconnected and dependent on each other, and this is reflected in the standard document. Effective instruction often incorporates several performance objectives into an integrated experience of learning for the student.

The content in College Work Readiness (grades 11-12) is a new addition to the Mathematics Standard. This content is separated into the five main strands. Performance objectives highlighted in italics in the document have been identified as core to an Algebra II course. As districts/schools create additional high school mathematics courses, they may select

from the comprehensive set of performance objectives contained within the five strands.

New to the 2008 Mathematics Standard is the development of more comprehensive grade level documents. The format of these documents will support the implementation of the revised standard. After each concept statement, there are summary expectations appropriate for that specific grade level. These statements provide a roadmap for instruction. Teachers will notice that there are now three columns of information. The first column lists the performance objectives with accompanying strand/concept and content area connections. The middle column highlights explicit connections to Strand 5, Concept 2 performance objectives. These performance objectives are grounded in the core processes of logic, reasoning, problem-solving and proof. The third column provides instructional support to teachers in the form of explanation and examples.

Strand One: Number and Operations

Number sense is the understanding of numbers and how they relate to each other and how they are used in specific context or real-world application. It includes an awareness of the different ways in which numbers are used, such as counting, measuring, labeling, and locating. It includes an awareness of the different types of numbers such as, whole numbers, integers, fractions, and decimals and the relationships between them and when each is most useful. Number sense includes an understanding of the size of numbers, so that students should be able to recognize that the volume of their room is closer to 1,000 than 10,000 cubic feet.

Students develop a sense of what numbers are, i.e., to use numbers and number relationships to acquire basic facts, to solve a wide variety of real-world problems, and to estimate to determine the reasonableness of results.

Concept 1: Number Sense

Understand and apply numbers, ways of representing numbers, the relationships among numbers, and different number systems.

Concept 2: Numerical Operations

Understand and apply numerical operations and their relationship to one another.

Concept 3: Estimation

Use estimation strategies reasonably and fluently while integrating content from each of the other strands.

Strand 2: Data Analysis, Probability, and Discrete Mathematics

This strand requires students to use data collection, data analysis, statistics, probability, systematic listing and counting, and the study of graphs. This prepares students for the study of discrete functions as well as to make valid inferences, decisions, and arguments.

Discrete mathematics is a branch of mathematics that is widely used in business and industry. Combinatorics is the mathematics of systematic counting. Vertex-edge graphs are

used to model and solve problems involving paths, networks, and relationships among a finite number of objects.

Concept 1: Data Analysis (Statistics)
Understand and apply data collection, organization, and representation to analyze and sort data. This is considered to be the analysis and interpretation of numerical data in terms of samples and populations.

Concept 2: Probability
Understand and apply the basic concepts of probability. This is the field of mathematics that deals with the likelihood that an event will occur expressed as the ratio of the number of favorable outcomes in the set of outcomes to the total number of possible outcomes.

Concept 3: Systematic Listing and Counting
Understand and demonstrate the systematic listing and counting of possible outcomes. This field of mathematics is generally referred to as Combinatorics.

Concept 4: Vertex-Edge Graphs
Understand and apply the concepts of vertex-edge graphs and networks. This field connects graph theory with practical problems.

Strand 3: Patterns, Algebra, and Functions

Patterns occur everywhere in nature. Algebraic methods are used to explore, model and describe patterns, relationships, and functions involving numbers, shapes, iteration, recursion, and graphs within a variety of real-world problem solving situations. Iteration and recursion are used to model sequential, step-by-step change.

Algebra emphasizes relationships among quantities, including functions, ways of representing mathematical relationships, and the analysis of change.

Concept 1: Patterns
Identify patterns and apply pattern recognition to reason mathematically. Students begin with simple repetitive patterns of many iterations. This is the beginning of recursive thinking. Later, students can study sequences that can best be defined using recursion.

Concept 2: Functions & Relationships
Describe and model functions and their relationships. For example, distribution and communication networks, laws of physics, population models, and statistical results can all be represented in the symbolic language of algebra.

Concept 3: Algebraic Representations
Represent and analyze mathematical situations and structures using algebraic representations. Algebraic representation is about abstract structures and about using the principles of those structures in solving problems expressed with symbols.

Concept 4: Analysis of Change
Analyze how changing the values of one quantity corresponds to change in the values of another quantity.

Mathematics Standard Articulated by Grade Level

Strand 4: Geometry and Measurement

Geometry is a natural place for the development of students' reasoning, higher thinking, and justification skills culminating in work with proofs. Geometric modeling and spatial reasoning offer ways to interpret and describe physical environments and can be important tools in problem solving. Students use geometric methods, properties and relationships, transformations, and coordinate geometry as a means to recognize, draw, describe, connect, analyze, and measure shapes and representations in the physical world.

Measurement is the assignment of a numerical value to an attribute of an object, such as the length of a pencil. At more sophisticated levels, measurement involves assigning a number to a characteristic of a situation, as is done by the consumer price index. A major emphasis in this strand is becoming familiar with the units and processes that are used in measuring attributes.

Concept 1: Geometric Properties
Analyze the attributes and properties of two- and three-dimensional figures and develop mathematical arguments about their relationships (in conjunction with strand 5, concept 2).

Concept 2: Transformation of Shapes
Apply spatial reasoning to create transformations and use symmetry to analyze mathematical situations.

Concept 3: Coordinate Geometry
Specify and describe spatial relationships using coordinate geometry and other representational systems.

Concept 4: Measurement
Understand and apply appropriate units of measure, measurement techniques, and formulas to determine measurements.

Strand 5: Structure and Logic

This strand emphasizes the core processes of problem solving. Students draw from the content of the other four strands to devise algorithms and analyze algorithmic thinking. Strand One and Strand Three provide the conceptual and computational basis for these algorithms. Logical reasoning and proof draws its substance from the study of geometry, patterns, and analysis to connect remaining strands. Students use algorithms, algorithmic thinking, and logical reasoning (both inductive and deductive) as they make conjectures and test the validity of arguments and proofs. Concept two develops the core processes as students evaluate situations, select problem solving strategies, draw logical conclusions, develop and describe solutions, and recognize their applications.

Concept 1: Algorithms and Algorithmic Thinking
Use reasoning to solve mathematical problems. Determine step-by-step series of instructions to explain mathematical processes.

Mathematics Standard Articulated by Grade Level

Concept 2: Logic, Reasoning, Problem Solving, and Proof
Evaluate situations, select problem solving strategies, draw logical conclusions, develop and describe solutions, and recognize their applications. Develop mathematical arguments based on induction and deduction, and distinguish between valid and invalid arguments.

Mathematics Standard Articulated by Grade Level

Strand 1: Number and Operations

Every student should understand and use all concepts and skills from the previous grade levels. The standard is designed so that new learning builds on preceding skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of all mathematical strands.

Concept 1: Number Sense	The concept of understanding and applying numbers, ways of representing numbers, and the relationships among numbers and different number systems.
Concept 2: Numerical Operations	The concept of understanding and applying numerical operations and their relationship to one another.
Concept 3: Estimation	The concept of using estimation strategies reasonably and fluently while integrating content from each of the other strands.

Strand 2: Data Analysis, Probability, and Discrete Mathematics

Every student should understand and use all concepts and skills from the previous grade levels. The standard is designed so that new learning builds on preceding skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Data Analysis (Statistics)	The concept of understanding and applying data collection, organization, and representation to analyze and sort data
Concept 2: Probability	The concept of understanding and applying the basic concepts of probability.
Concept 3: Systematic Listing & Counting	The concept of understanding and demonstrating the systematic listing and counting of possible outcomes.
Concept 4: Vertex-Edge Graphs	The concept of understanding and applying vertex-edge graphs.

Mathematics Standard Articulated by Grade Level

Strand 3: Patterns, Algebra, and Functions

Every student should understand and use all concepts and skills from the previous grade levels. The standard is designed so that new learning builds on preceding skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of all mathematical strands.

Concept 1: Patterns	The concept of identifying patterns and applying pattern recognition to reason mathematically while integrating the content from each of the other strands.
Concept 2: Functions and Relationships	The concept of describing and modeling functions and their relationships.
Concept 3: Algebraic Representations	The concept of representing and analyzing mathematical situations and structures using algebraic representations.
Concept 4: Analysis of Change	The concept of analyzing how changing the values of one quantity corresponds to change in the values of another quantity.

Strand 4: Geometry and Measurement

Every student should understand and use all concepts and skills from the previous grade levels. The standard is designed so that new learning builds on preceding skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of all mathematical strands.

Concept 1: Geometric Properties	The concept of analyzing the attributes and properties of two- and three- dimensional figures and developing mathematical arguments about their relationships.
Concept 2: Transformation of Shapes	The concept of applying spatial reasoning to create transformations and using symmetry to analyze mathematical situations.
Concept 3: Coordinate Geometry	The concept of specifying and describing spatial relationships using rectangular and other coordinate systems while integrating content from each of the other strands.
Concept 4: Measurement	The concept of understanding and applying appropriate units of measure, measurement techniques, and formulas to determine measurements.

Mathematics Standard Articulated by Grade Level

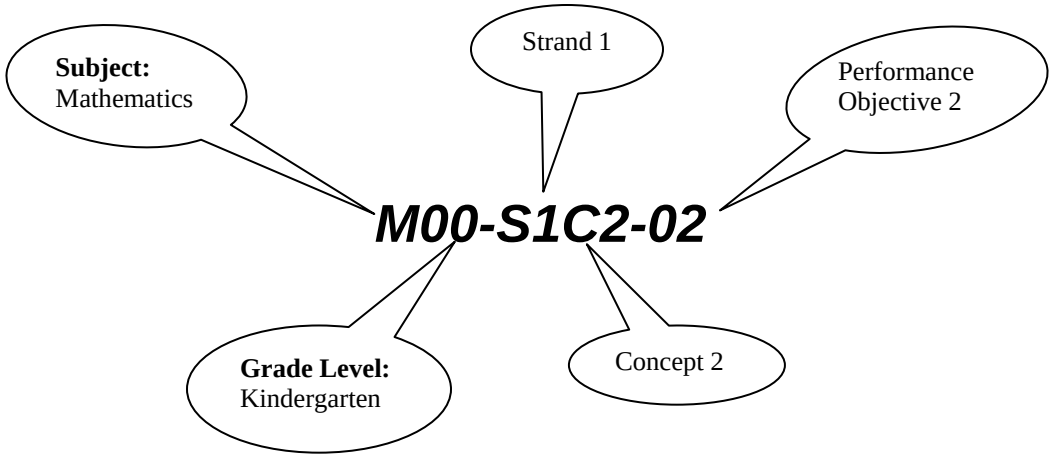
Strand 5: Structure and Logic

Every student should understand and use all concepts and skills from the previous grade levels. The standard is designed so that new learning builds on preceding skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of all mathematical strands.

Concept 1: Algorithms and Algorithmic Thinking	The concept of using reasoning to solve mathematical problems.
Concept 2: Logic, Reasoning, Problem Solving and Proof	The concept of evaluating situations, selecting problem-solving strategies, drawing logical conclusions, developing and describing solutions and recognizing their applications.

Mathematics Standard Articulated by Grade Level

Coding for Articulated Standards



Examples of Mathematics items:

M04-S3C1-03 (Grade 4, Strand 3, Concept 1, PO 3)
MHS-S2C2-01 (High School, Strand 2, Concept 2, PO 1)
MCWR-S5C1-03 (College Work Readiness, Strand 5, Concept 1, PO 3)

Connections are provided in the Mathematics Standard where appropriate in the grade level documents. Connections to the Language Arts Standards are embedded throughout the Mathematics Standard so those connections are not explicitly listed. Examples of coding for other subjects are shown below:

Examples of Science items:

SC01-S1C2-02 (Grade 1, Strand 1, Concept 2, PO 2)
SCHS-S5C1-01 (High School, Strand 5, Concept 1, PO 1)

Examples of Social Studies items:

SS01-S1C2-02 (Grade 1, Strand 1, Concept 2, PO 2)
SSHS-S5C1-01 (High School, Strand 5, Concept 1, PO 1)

Mathematics Standard Articulated by Grade Level
Strand 1: Number and Operations

Concept 1: Number Sense Understand and apply numbers, ways of representing numbers, and the relationships among numbers and different number systems.						
Kindergarten	**Grade 1**	**Grade 2**	**Grade 3**	**Grade 4**	**Grade 5**	**Grade 6**
PO 1. Express whole numbers 0 to 20 using and connecting multiple representations.	PO 1. Express whole numbers 0 to 100, in groups of tens and ones using and connecting multiple representations.	PO 1. Express whole numbers 0 to 1000, in groups of hundreds, tens and ones using and connecting multiple representations.	PO 1. Express whole numbers through six digits using and connecting multiple representations.	PO 1. Express whole numbers, fractions, decimals, and percents using and connecting multiple representations.	PO 1. Determine equivalence by converting between benchmark fractions, decimals, and percents.	PO 1. Convert between expressions for positive rational numbers, including fractions, decimals, percents, and ratios.
PO 2. Count forward to 20 and backward from 10 with or without objects using different starting points.	PO 2. Count forward to 100 and backward from 100 by 1s and 10s using different starting points, and count forward to 100 by 2s and 5s.	PO 2. Count forward to 1000 and backward from 1000 by 1s, 10s, and 100s using different starting points.				
PO 3. Identify numbers which are one more or less than a given number to 20.	PO 3. Identify numbers which are 10 more or less than a given number to 90.	PO 3. Identify numbers which are 100 more or less than a given number to 900.				
PO 4. Compare and order whole numbers through 20.	PO 4. Compare and order whole numbers through 100 by applying the concepts of place value.	PO 4. Compare and order whole numbers through 1000 by applying the concept of place value.	PO 2. Compare and order whole numbers through six digits by applying the concept of place value.			
PO 5. Recognize and compare the ordinal position of at least five objects.	PO 5. Recognize and compare ordinal numbers, first through tenth.					
		PO 5. Count money to $1.00.	PO 3. Count and represent money using coins and bills to $100.00.			

The bulleted items within a performance objective indicate the specific content to be taught.

Mathematics Standard Articulated by Grade Level
Strand 1: Number and Operations

Concept 1: Number Sense Understand and apply numbers, ways of representing numbers, and the relationships among numbers and different number systems.						
Kindergarten	**Grade 1**	**Grade 2**	**Grade 3**	**Grade 4**	**Grade 5**	**Grade 6**
		PO 6. Sort whole numbers through 1000 into odd and even, and justify the sort.	PO 4. Sort whole numbers into sets and justify the sort.	PO 2. Compose and decompose whole numbers using factors and multiples.	PO 2. Differentiate between prime and composite numbers; differentiate between factors and multiples for whole numbers.	PO 2. Use prime factorization to • express a whole number as a product of its prime factors and • determine the greatest common factor and least common multiple of two whole numbers.
			PO 5. Express benchmark fractions as fair sharing, parts of a whole, or parts of a set.	PO 3. Express fractions as fair sharing, parts of a whole, parts of a set, and locations on a real number line.	PO 3. Locate integers on a number line.	PO 3. Demonstrate an understanding of fractions as rates, division of whole numbers, parts of a whole, parts of a set, and locations on a real number line.
			PO 6. Compare and order benchmark fractions.	PO 4. Compare and order decimals to hundredths.	PO 4. Compare and order positive fractions, decimals, and percents.	PO 4. Compare and order integers; and positive fractions, decimals, and percents.
				PO 5. Use simple ratios to describe problems in context.	PO 5. Use ratios and unit rates to model, describe and extend problems in context.	
					PO 6. Express or interpret positive and negative numbers in context.	

The bulleted items within a performance objective indicate the specific content to be taught.

Mathematics Standard Articulated by Grade Level
Strand 1: Number and Operations

Concept 1: Number Sense Understand and apply numbers, ways of representing numbers, and the relationships among numbers and different number systems.						
Kindergarten	**Grade 1**	**Grade 2**	**Grade 3**	**Grade 4**	**Grade 5**	**Grade 6**
						PO 5. Express that a number's distance from zero on the number line is its absolute value.
						PO 6. Express the inverse relationships between exponents and roots for perfect squares and cubes.

The bulleted items within a performance objective indicate the specific content to be taught.

Mathematics Standard Articulated by Grade Level
Strand 1: Number and Operations

Concept 2: Numerical Operations Understand and apply numerical operations and their relationship to one another.						
Kindergarten	**Grade 1**	**Grade 2**	**Grade 3**	**Grade 4**	**Grade 5**	**Grade 6**
PO 1. Solve contextual problems by developing, applying, and recording strategies with sums and minuends to 10 using objects, pictures, and symbols.	PO 1. Solve contextual problems using multiple representations for addition and subtraction facts.	PO 1. Solve contextual problems using multiple representations involving • addition and subtraction with one- and/or two-digit numbers, • multiplication for 1s, 2s, 5s, and 10s, and • adding and subtracting money to $1.00.				
PO 2. Develop and use multiple strategies to determine • sums to 10 and • differences with minuends to 10.	PO 2. Demonstrate addition and subtraction of numbers that total less than 100 by using various representations that connect to place value concepts.	PO 2. Demonstrate the ability to add and subtract whole numbers (to two digits) and decimals (in the context of money) • with up to three addends and • to $1.00.	PO 1. Add and subtract whole numbers to four digits.	PO 1. Add and subtract decimals through hundredths including money to $1000.00 and fractions with like denominators.	PO 1. Add and subtract decimals through thousandths and fractions expressing solutions in simplest form.	
	PO 3. Develop and use multiple strategies for addition facts to 10+10 and their related subtraction facts.	PO 3. Demonstrate fluency of addition and subtraction facts.				

The bulleted items within a performance objective indicate the specific content to be taught.

Mathematics Standard Articulated by Grade Level
Strand 1: Number and Operations

Concept 2: Numerical Operations Understand and apply numerical operations and their relationship to one another.						
Kindergarten	**Grade 1**	**Grade 2**	**Grade 3**	**Grade 4**	**Grade 5**	**Grade 6**
		PO 4. Apply and interpret the concept of addition and subtraction as inverse operations to solve problems.				PO 1. Apply and interpret the concepts of addition and subtraction with integers using models.
PO 3. Create word problems based on sums to 10 and differences with minuends to 10.	PO 4. Create word problems based on addition and subtraction facts.	PO 5. Create and solve word problems based on addition and subtraction of two-digit numbers.	PO 2. Create and solve word problems based on addition, subtraction, multiplication, and division.			
		PO 6. Demonstrate the concept of multiplication for 1s, 2s, 5s, and 10s.	PO 3. Demonstrate the concept of multiplication and division using multiple models.	PO 2. Use multiple strategies to multiply whole numbers • two-digit by two-digit and • multi-digit by one-digit.	PO 2. Multiply multi-digit whole numbers.	PO 2. Multiply multi-digit decimals through thousandths.
			PO 4. Demonstrate fluency of multiplication and division facts through 10.	PO 3. Demonstrate fluency of multiplication and division facts through 12.		
			PO 5. Apply and interpret the concept of multiplication and division as inverse operations to solve problems.	PO 4. Use multiple strategies to divide whole numbers.	PO 3. Divide multi-digit whole numbers by whole number divisors with and without remainders.	PO 3. Divide multi-digit whole numbers and decimals by decimal divisors with and without remainders.
		PO 7. Describe the effect of operations (addition and subtraction) on the size of whole numbers.	PO 6. Describe the effect of operations (multiplication and division) on the size of whole numbers.			

The bulleted items within a performance objective indicate the specific content to be taught.

Mathematics Standard Articulated by Grade Level
Strand 1: Number and Operations

Concept 2: Numerical Operations Understand and apply numerical operations and their relationship to one another.						
Kindergarten	**Grade 1**	**Grade 2**	**Grade 3**	**Grade 4**	**Grade 5**	**Grade 6**
						PO 4. Multiply and divide fractions.
						PO 5. Provide a mathematical argument to explain operations with two or more fractions or decimals.
	PO 5. Apply properties to solve addition/subtraction problems • identity property of addition/subtraction and • commutative property of addition.	PO 8. Apply properties to solve addition/subtraction problems • identity property of addition/ subtraction, • commutative property of addition, and • associative property of addition.	PO 7. Apply commutative, identity, and zero properties to multiplication and apply the identity property to division.	PO 5. Apply associative and distributive properties to solve multiplication and division problems.	PO 4. Apply the associative, commutative, and distributive properties to solve numerical problems.	PO 6. Apply the commutative, associative, distributive, and identity properties to evaluate numerical expressions involving whole numbers.
				PO 6. Apply order of operations with whole numbers.	PO 5. Simplify numerical expressions (including fractions and decimals) using the order of operations with or without grouping symbols.	PO 7. Simplify numerical expressions (involving fractions, decimals, and exponents) using the order of operations with or without grouping symbols.

The bulleted items within a performance objective indicate the specific content to be taught.

Mathematics Standard Articulated by Grade Level
Strand 1: Number and Operations

Concept 3: Estimation Use estimation strategies reasonably and fluently while integrating content from each of the other strands.						
Kindergarten	**Grade 1**	**Grade 2**	**Grade 3**	**Grade 4**	**Grade 5**	**Grade 6**
PO 1. Identify quantities to 20 as more or less than 5 or as more or less than 10.	PO 1. Use estimation to determine if sums are more or less than 5, more or less than 10, or more or less than 20.	PO 1. Use estimation to determine if sums of two 2-digit numbers are more or less than 20, more or less than 50, or more or less than 100.		PO 1. Use benchmarks as meaningful points of comparison for whole numbers, decimals, and fractions.		PO 1. Use benchmarks as meaningful points of comparison for rational numbers.
			PO 1. Make estimates appropriate to a given situation or computation with whole numbers.	PO 2. Make estimates appropriate to a given situation or computation with whole numbers and fractions.	PO 1. Make estimates appropriate to a given situation or computation with whole numbers, fractions, and decimals.	PO 2. Make estimates appropriate to a given situation and verify the reasonableness of the results.

The bulleted items within a performance objective indicate the specific content to be taught.

Mathematics Standard Articulated by Grade Level
Strand 2: Data Analysis, Probability, and Discrete Mathematics

Concept 1: Data Analysis (Statistics) Understand and apply data collection, organization, and representation to analyze and sort data.						
Kindergarten	**Grade 1**	**Grade 2**	**Grade 3**	**Grade 4**	**Grade 5**	**Grade 6**
PO 1. Construct simple displays of data using objects or pictures.	PO 1. Collect, record, organize, and display data using tally charts or pictographs.	PO 1. Collect, record, organize, and display data using pictographs, frequency tables, or single bar graphs.	PO 1. Collect, record, organize, and display data using frequency tables, single bar graphs, or single line graphs.	PO 1. Collect, record, organize, and display data using double bar graphs, single line graphs, or circle graphs.	PO 1. Collect, record, organize, and display data using multi-bar graphs or double line graphs.	PO 1. Solve problems by selecting, constructing, and interpreting displays of data, including histograms and stem-and-leaf plots.
PO 2. Ask and answer questions by counting, comparing quantities, and interpreting simple displays of data.	PO 2. Ask and answer questions by interpreting simple displays of data, including tally charts or pictographs.	PO 2. Formulate and answer questions by interpreting displays of data, including pictographs, frequency tables, or single bar graphs.	PO 2. Formulate and answer questions by interpreting and analyzing displays of data, including frequency tables, single bar graphs, or single line graphs.	PO 2. Formulate and answer questions by interpreting and analyzing displays of data, including double bar graphs, single line graphs, or circle graphs.	PO 2. Formulate and answer questions by interpreting and analyzing displays of data, including multi-bar graphs or double line graphs.	PO 2. Formulate and answer questions by interpreting, analyzing, and drawing inferences from displays of data, including histograms and stem-and-leaf plots.
				PO 3. Use median, mode, and range to describe the distribution of a given data set.	PO 3. Use mean, median, mode, and range to analyze and describe the distribution of a given data set.	PO 3. Use extreme values, mean, median, mode, and range to analyze and describe the distribution of a given data set.
				PO 4. Compare two sets of related data.		PO 4. Compare two or more sets of data by identifying trends.

The bulleted items within a performance objective indicate the specific content to be taught.

Mathematics Standard Articulated by Grade Level
Strand 2: Data Analysis, Probability, and Discrete Mathematics

Concept 2: Probability Understand and apply the basic concepts of probability.						
Kindergarten	**Grade 1**	**Grade 2**	**Grade 3**	**Grade 4**	**Grade 5**	**Grade 6**
				PO 1. Describe elements of theoretical probability by listing or drawing all possible outcomes of a given event and predicting the outcome using word and number benchmarks.	PO 1. Describe the theoretical probability of events and represent the probability as a fraction, decimal, or percent.	PO 1. Use data collected from multiple trials of a single event to form a conjecture about the theoretical probability.
					PO 2. Explore probability when performing experiments by • predicting the outcome, • recording the data, • comparing outcomes of the experiment to predictions, and • comparing the results of multiple repetitions of the experiment.	PO 2. Use theoretical probability to • predict experimental outcomes, • compare the outcome of the experiment to the prediction, and • replicate the experiment and compare results.
						PO 3. Determine all possible outcomes (sample space) of a given situation using a systematic approach.

The bulleted items within a performance objective indicate the specific content to be taught.

Mathematics Standard Articulated by Grade Level
Strand 2: Data Analysis, Probability, and Discrete Mathematics

Concept 3: Systematic Listing and Counting Understand and demonstrate the systematic listing and counting of possible outcomes.						
Kindergarten	**Grade 1**	**Grade 2**	**Grade 3**	**Grade 4**	**Grade 5**	**Grade 6**
PO 1. Sort, classify, count, and represent up to 20 objects and justify the sorting rule.	PO 1. Use Venn diagrams to sort, classify, and count objects and justify the sorting rule.	PO 1. List all possibilities in counting situations.	PO 1. Represent all possibilities for a variety of counting problems using arrays, charts, and systematic lists; draw conclusions from these representations.	PO 1. Construct tree diagrams to solve problems in context by • representing all possibilities for a variety of counting problems, • explaining how its properties relate to the problem, • representing the same counting problem in multiple ways, and • drawing conclusions.	PO 1. Analyze relationships among representations and make connections to the multiplication principle of counting.	PO 1. Build and explore tree diagrams where items repeat.
		PO 2. Solve a variety of problems based on the addition principle of counting.	PO 2. Solve a variety of problems based on the multiplication principle of counting.	PO 2. Justify that all possibilities have been enumerated without duplication.	PO 2. Solve a variety of counting problems and explain the multiplication principle of counting.	PO 2. Explore counting problems with Venn diagrams using three attributes.

The bulleted items within a performance objective indicate the specific content to be taught.

Mathematics Standard Articulated by Grade Level
Strand 2: Data Analysis, Probability, and Discrete Mathematics

Concept 4: Vertex-Edge Graphs Understand and apply vertex-edge graphs.						
Kindergarten	**Grade 1**	**Grade 2**	**Grade 3**	**Grade 4**	**Grade 5**	**Grade 6**
		PO 1. Color simple pictures or maps using the least number of colors and justify the coloring.	PO 1. Color complex maps using the least number of colors and justify the coloring.	PO 1. Demonstrate the connection between map coloring and vertex coloring.		
		PO 2. Build vertex-edge graphs using concrete materials and explore properties of vertex-edge graphs • number of vertices and edges, • neighboring vertices, and • paths in a graph.	PO 2. Investigate properties of vertex-edge graphs • circuits in a graph, • weights on edges, and • shortest path between two vertices.	PO 2. Construct vertex-edge graphs to represent concrete situations and identify paths and circuits.	PO 1. Investigate properties of vertex-edge graphs • Euler paths, • Euler circuits, and • degree of a vertex.	PO 1. Investigate properties of vertex-edge graphs • Hamilton paths, • Hamilton circuits, and • shortest route.
		PO 3. Construct simple vertex-edge graphs from simple pictures or maps.	PO 3. Solve problems using vertex-edge graphs.	PO 3. Solve conflict problems by constructing and coloring vertex-edge graphs.	PO 2. Solve problems related to Euler paths and circuits.	PO 2. Solve problems related to Hamilton paths and circuits.

The bulleted items within a performance objective indicate the specific content to be taught.

Mathematics Standard Articulated by Grade Level
Strand 3: Patterns, Algebra, and Functions

Concept 1: Patterns Identify patterns and apply pattern recognition to reason mathematically while integrating content from each of the other strands.						
Kindergarten	**Grade 1**	**Grade 2**	**Grade 3**	**Grade 4**	**Grade 5**	**Grade 6**
PO 1. Recognize, describe, extend, create, and record simple repeating patterns.	PO 1. Recognize, describe, extend, create, and record repeating patterns.	PO 1. Recognize, describe, extend, create, and find missing terms in a numerical or symbolic pattern.	PO 1. Recognize, describe, extend, create, and find missing terms in a numerical sequence.	PO 1. Recognize, describe, create, extend, and find missing terms in a numerical sequence involving whole numbers using all four basic operations.	PO 1. Recognize, describe, create, and analyze a numerical sequence involving fractions and decimals using addition and subtraction.	PO 1. Recognize, describe, create, and analyze a numerical sequence involving fractions and decimals using all four basic operations.
PO 2. Recognize, describe, extend, and record simple growing patterns.	PO 2. Recognize, describe, extend, create, and record growing patterns.					
		PO 2. Explain the rule for a given numerical or symbolic pattern and verify that the rule works.	PO 2. Explain the rule for a given numerical sequence and verify that the rule works.	PO 2. Explain the rule for a given numerical sequence, verify that the rule works, and use the rule to make predictions.		

The bulleted items within a performance objective indicate the specific content to be taught.

Mathematics Standard Articulated by Grade Level
Strand 3: Patterns, Algebra, and Functions

Concept 2: Functions and Relationships Describe and model functions and their relationships.						
Kindergarten	**Grade 1**	**Grade 2**	**Grade 3**	**Grade 4**	**Grade 5**	**Grade 6**
		PO 1. Describe a rule that represents a given relationship between two quantities using words or pictures.	PO 1. Recognize and describe a relationship between two quantities, given by a chart, table, or graph, in which the quantities change proportionally, using words, pictures, or expressions.			PO 1. Recognize and describe a relationship between two quantities, given by a chart, table, or graph, using words and expressions.
			PO 2. Translate between the different representations of whole number relationships, including symbolic, numerical, verbal, or pictorial.			

The bulleted items within a performance objective indicate the specific content to be taught.

Mathematics Standard Articulated by Grade Level
Strand 3: Patterns, Algebra, and Functions

Concept 3: Algebraic Representations Represent and analyze mathematical situations and structures using algebraic representations.						
Kindergarten	**Grade 1**	**Grade 2**	**Grade 3**	**Grade 4**	**Grade 5**	**Grade 6**
PO 1. Record equivalent forms of whole numbers to 10 by constructing models and using numbers.	PO 1. Record equivalent forms of whole numbers to 100 by constructing models and using numbers.	PO 1. Record equivalent forms of whole numbers to 1000 by constructing models and using numbers.	PO 1. Record equivalent forms of whole numbers to six digits by constructing models and using numbers.			
PO 2. Compare expressions using spoken words and the symbol =.	PO 2. Compare expressions using spoken words and the symbols = and ≠.	PO 2. Compare expressions using spoken words and the symbols =, ≠, <, and >.				
	PO 3. Represent a word problem requiring addition or subtraction facts using an equation.	PO 3. Represent a word problem requiring addition or subtraction through 100 using an equation.				
		PO 4. Identify the value of an unknown number in an equation involving an addition or subtraction fact.	PO 2. Use a symbol to represent an unknown quantity in a given context.	PO 1. Use a symbol to represent an unknown quantity in a simple algebraic expression involving all operations.		PO 1. Use an algebraic expression to represent a quantity in a given context.
			PO 3. Create and solve simple one-step equations that can be solved using addition and multiplication facts.	PO 2. Create and solve one-step equations that can be solved using addition, subtraction, multiplication, and division of whole numbers.	PO 1. Create and solve two-step equations that can be solved using inverse operations with whole numbers.	PO 2. Create and solve two-step equations that can be solved using inverse properties with fractions and decimals.
						PO 3. Translate both ways between a verbal description and an algebraic expression or equation.

The bulleted items within a performance objective indicate the specific content to be taught.

Mathematics Standard Articulated by Grade Level
Strand 3: Patterns, Algebra, and Functions

Concept 3: Algebraic Representations Represent and analyze mathematical situations and structures using algebraic representations.						
Kindergarten	**Grade 1**	**Grade 2**	**Grade 3**	**Grade 4**	**Grade 5**	**Grade 6**
						PO 4. Evaluate an expression involving the four basic operations by substituting given fractions and decimals for the variable.

Concept 4: Analysis of Change Analyze how changing the values of one quantity corresponds to change in the values of another quantity.						
Kindergarten	**Grade 1**	**Grade 2**	**Grade 3**	**Grade 4**	**Grade 5**	**Grade 6**
				PO 1. Identify the change in a quantity over time and make simple predictions.	PO 1. Describe patterns of change including constant rate and increasing or decreasing rate.	PO 1. Determine a pattern to predict missing values on a line graph or scatterplot.

The bulleted items within a performance objective indicate the specific content to be taught.

Mathematics Standard Articulated by Grade Level
Strand 4: Geometry and Measurement

Concept 1: Geometric Properties Analyze the attributes and properties of 2- and 3- dimensional figures and develop mathematical arguments about their relationships.						
Kindergarten	**Grade 1**	**Grade 2**	**Grade 3**	**Grade 4**	**Grade 5**	**Grade 6**
PO 1. Identify, analyze, and describe circles, triangles, and rectangles (including squares) in different orientations and environments.	PO 1. Identify and draw 2-dimensional geometric figures based on given attributes regardless of size or orientation.			PO 1. Draw and describe the relationships between points, lines, line segments, rays, and angles including parallelism and perpendicularity.	PO 1. Draw and label 2-dimensional figures given specific attributes including angle measure and side length.	
PO 2. Build, draw, compare, describe, and sort 2-dimensional figures (including irregular figures) using attributes.	PO 2. Compare and sort basic 2-dimensional figures (including irregular figures) using attributes and explain the reasoning for the sorting.	PO 1. Describe and compare the attributes of polygons up to six sides using the terms side, vertex, point, and length.	PO 1. Describe sequences of 2-dimensional figures created by increasing the number of sides, changing size, or changing orientation.	PO 2. Justify which objects in a collection match a given geometric description.	PO 2. Solve problems by understanding and applying the property that the sum of the interior angles of a triangle is 180˚.	PO 1. Define π (pi) as the ratio between the circumference and diameter of a circle and explain the relationship among the diameter, radius, and circumference.
				PO 3. Describe and classify triangles by angles and sides.	PO 3. Classify quadrilaterals by their properties.	
	PO 3. Describe the results of composing and decomposing 2-dimensional figures.			PO 4. Recognize which attributes (such as shape or area) change and which do not change when 2-dimensional figures are cut up or rearranged.		
			PO 2. Recognize similar figures.	PO 5. Recognize and draw congruent figures, and match them in a given collection.		

The bulleted items within a performance objective indicate the specific content to be taught.

Mathematics Standard Articulated by Grade Level
Strand 4: Geometry and Measurement

Concept 1: Geometric Properties Analyze the attributes and properties of 2- and 3- dimensional figures and develop mathematical arguments about their relationships.						
Kindergarten	**Grade 1**	**Grade 2**	**Grade 3**	**Grade 4**	**Grade 5**	**Grade 6**
				PO 6. Draw right, acute, obtuse, and straight angles and identify these angles in other geometric figures.		PO 2. Solve problems using properties of supplementary, complementary, and vertical angles.
			PO 3. Identify and describe 3-dimensional figures including their relationship to real world objects: sphere, cube, cone, cylinder, pyramids, and rectangular prisms.	PO 7. Recognize the relationship between a 3-dimensional figure and its corresponding net(s).		
			PO 4. Describe and compare attributes of two- and three-dimensional figures.		PO 4. Compare attributes of 2-dimensional figures with 3-dimensional figures by drawing and constructing nets and models.	

The bulleted items within a performance objective indicate the specific content to be taught.

Mathematics Standard Articulated by Grade Level
Strand 4: Geometry and Measurement

Concept 2: Transformation of Shapes Apply spatial reasoning to create transformations and use symmetry to analyze mathematical situations.						
Kindergarten	**Grade 1**	**Grade 2**	**Grade 3**	**Grade 4**	**Grade 5**	**Grade 6**
			PO 1. Identify a translation, reflection, or rotation and model its effect on a 2-dimensional figure.			PO 1. Identify a simple translation or reflection and model its effect on a 2-dimensional figure on a coordinate plane using all four quadrants.
		PO 1. Identify, with justification, whether a 2-dimensional figure has lines of symmetry.	PO 2. Identify, with justification, all lines of symmetry in a 2-dimensional figure.			PO 2. Draw a reflection of a polygon in the coordinate plane using a horizontal or vertical line of reflection.

The bulleted items within a performance objective indicate the specific content to be taught.

Mathematics Standard Articulated by Grade Level
Strand 4: Geometry and Measurement

Concept 3: Coordinate Geometry Specify and describe spatial relationships using rectangular and other coordinate systems while integrating content from each of the other strands.						
Kindergarten	**Grade 1**	**Grade 2**	**Grade 3**	**Grade 4**	**Grade 5**	**Grade 6**
				PO 1. Name, locate, and graph points in the first quadrant of the coordinate plane using ordered pairs.		PO 1. Graph ordered pairs in any quadrant of the coordinate plane.
				PO 2. Plot line segments in the first quadrant of the coordinate plane using a set of ordered pairs in a table.		
				PO 3. Construct geometric figures with vertices at points on the coordinate plane.		PO 2. State the missing coordinate of a given figure on the coordinate plane using geometric properties to justify the solution.

The bulleted items within a performance objective indicate the specific content to be taught.

Mathematics Standard Articulated by Grade Level
Strand 4: Geometry and Measurement

Concept 4: Measurement Understand and apply appropriate units of measure, measurement techniques, and formulas to determine measurements.						
Kindergarten	**Grade 1**	**Grade 2**	**Grade 3**	**Grade 4**	**Grade 5**	**Grade 6**
PO 1. Compare and order objects according to observable and measureable attributes.	PO 1. Compare and order objects according to length, capacity, and weight.					
PO 2. Use the attribute of length to describe and compare objects using non-standard units.	PO 2. Measure and compare the length of objects using the benchmark of one inch.					
	PO 3. Sequence the days of the week and the months of the year.	PO 1. Tell time to the nearest minute using analog and digital clocks.	PO 1. Determine elapsed time • across months using a calendar • by hours and half hours using a clock.	PO 1. Compute elapsed time to the minute.	PO 1. Solve problems using elapsed time.	
		PO 2. Apply measurement skills to measure the attributes of an object (length, capacity, weight).	PO 2. Apply measurement skills to measure length, weight, and capacity using US Customary units.	PO 2. Apply measurement skills to measure length, mass, and capacity using metric units.	PO 2. State an appropriate measure and degree of accuracy in a given context.	PO 1. Determine the appropriate unit of measure for a given context and the appropriate tool to measure to the needed precision (including length, capacity, angles, time, and mass).
		PO 3. Read temperatures on a thermometer using Fahrenheit and Celsius.				
					PO 3. Measure angles between 0 and 360 degrees.	

The bulleted items within a performance objective indicate the specific content to be taught.

Mathematics Standard Articulated by Grade Level
Strand 4: Geometry and Measurement

Concept 4: Measurement

Understand and apply appropriate units of measure, measurement techniques, and formulas to determine measurements.

Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4	Grade 5	Grade 6
		PO 4. Demonstrate unit conversions • 1 foot = 12 inches, • 1 quart = 4 cups, • 1 pound = 16 ounces, • 1 hour = 60 minutes, • 1 day = 24 hours, • 1 week = 7 days, and • 1 year = 12 months.	PO 3. Convert units of length, weight, and capacity • inches or feet to yards, • ounces to pounds, and • cups to pints, pints to quarts, quarts to gallons.	PO 3. Solve problems involving conversions within the same measurement system.		PO 2. Solve problems involving conversion within the U.S. Customary and within the metric system.
						PO 3. Estimate the measure of objects using a scale drawing or map.
			PO 4. Determine the area of a rectangular figure using an array model.		PO 4. Solve problems involving the area of 2-dimensional figures by using the properties of parallelograms and triangles.	PO 4. Solve problems involving the area of simple polygons using formulas for rectangles and triangles.
			PO 5. Measure and calculate perimeter of 2-dimensional figures.	PO 4. Solve problems involving perimeter of 2-dimensional figures and area of rectangles.	PO 5. Solve problems involving area and perimeter of regular and irregular polygons using reallotment of square units.	PO 5. Solve problems involving area and perimeter of regular and irregular polygons.
				PO 5. Describe the change in perimeter or area when one attribute (length or width) of a rectangle changes.		

The bulleted items within a performance objective indicate the specific content to be taught.

Mathematics Standard Articulated by Grade Level
Strand 4: Geometry and Measurement

Concept 4: Measurement Understand and apply appropriate units of measure, measurement techniques, and formulas to determine measurements.						
Kindergarten	**Grade 1**	**Grade 2**	**Grade 3**	**Grade 4**	**Grade 5**	**Grade 6**
						PO 6. Describe the relationship between the volume of a figure and the area of its base.

The bulleted items within a performance objective indicate the specific content to be taught.

Mathematics Standard Articulated by Grade Level
Strand 5: Structure and Logic

Concept 1: Algorithms and Algorithmic Thinking Use reasoning to solve mathematical problems.						
Kindergarten	**Grade 1**	**Grade 2**	**Grade 3**	**Grade 4**	**Grade 5**	**Grade 6**
				PO 1. Analyze common algorithms for computing (adding, subtracting, multiplying, and dividing) with whole numbers using the associative, commutative, and distributive properties.	PO 1. Analyze common algorithms for adding and subtracting fractions and decimals using the associative, commutative, and distributive properties.	PO 1. Analyze algorithms for multiplying and dividing fractions and decimals using the associative, commutative, and distributive properties.
					PO 2. Develop an algorithm or formula to calculate areas and perimeters of simple polygons.	PO 2. Create and justify an algorithm to determine the area of a given compound figure using parallelograms and triangles.

The bulleted items within a performance objective indicate the specific content to be taught.

Mathematics Standard Articulated by Grade Level
Strand 5: Structure and Logic

Concept 2: Logic, Reasoning, Problem Solving, and Proof Evaluate situations, select problem-solving strategies, draw logical conclusions, develop and describe solutions, and recognize their applications.						
Kindergarten	**Grade 1**	**Grade 2**	**Grade 3**	**Grade 4**	**Grade 5**	**Grade 6**
PO 1. Identify the question(s) asked and any other questions that need to be answered in order to find a solution.	PO 1. Identify the question(s) asked and any other questions that need to be answered in order to find a solution.	PO 1. Identify the question(s) asked and any other questions that need to be answered in order to find a solution.	PO 1. Analyze a problem situation to determine the question(s) to be answered.	PO 1. Analyze a problem situation to determine the question(s) to be answered.	PO 1. Analyze a problem situation to determine the question(s) to be answered.	PO 1. Analyze a problem situation to determine the question(s) to be answered.
PO 2. Identify the given information that can be used to find a solution.	PO 2. Identify the given information that can be used to find a solution.	PO 2. Identify the given information that can be used to find a solution.	PO 2. Identify relevant, missing, and extraneous information related to the solution to a problem.	PO 2. Identify relevant, missing, and extraneous information related to the solution to a problem.	PO 2. Identify relevant, missing, and extraneous information related to the solution to a problem.	PO 2. Identify relevant, missing, and extraneous information related to the solution to a problem.
PO 3. Select from a variety of problem-solving strategies and use one or more strategies to arrive at a solution.	PO 3. Select from a variety of problem-solving strategies and use one or more strategies to arrive at a solution.	PO 3. Select from a variety of problem-solving strategies and use one or more strategies to arrive at a solution.	PO 3. Select and use one or more strategies to efficiently solve the problem and justify the selection.	PO 3. Select and use one or more strategies to efficiently solve the problem and justify the selection.	PO 3. Select and use one or more strategies to efficiently solve the problem and justify the selection.	PO 3. Analyze and compare mathematical strategies for efficient problem solving; select and use one or more strategies to solve a problem.
			PO 4. Determine whether a problem to be solved is similar to previously solved problems, and identify possible strategies for solving the problem.	PO 4. Determine whether a problem to be solved is similar to previously solved problems, and identify possible strategies for solving the problem.	PO 4. Determine whether a problem to be solved is similar to previously solved problems, and identify possible strategies for solving the problem.	PO 4. Apply a previously used problem-solving strategy in a new context.

The bulleted items within a performance objective indicate the specific content to be taught.

Mathematics Standard Articulated by Grade Level
Strand 5: Structure and Logic

Concept 2: Logic, Reasoning, Problem Solving, and Proof Evaluate situations, select problem-solving strategies, draw logical conclusions, develop and describe solutions, and recognize their applications.						
Kindergarten	**Grade 1**	**Grade 2**	**Grade 3**	**Grade 4**	**Grade 5**	**Grade 6**
PO 4. Represent a problem situation using any combination of words, numbers, pictures, physical objects, or symbols.	PO 4. Represent a problem situation using any combination of words, numbers, pictures, physical objects, or symbols.	PO 4. Represent a problem situation using any combination of words, numbers, pictures, physical objects, or symbols.	PO 5. Represent a problem situation using any combination of words, numbers, pictures, physical objects, or symbols.	PO 5. Represent a problem situation using any combination of words, numbers, pictures, physical objects, or symbols.	PO 5. Represent a problem situation using any combination of words, numbers, pictures, physical objects, or symbols.	PO 5. Represent a problem situation using multiple representations, describe the process used to solve the problem, and verify the reasonableness of the solution.
						PO 6. Communicate the answer(s) to the question(s) in a problem using appropriate representations, including symbols and informal and formal mathematical language.
PO 5. Explain and clarify mathematical thinking.	PO 5. Explain and clarify mathematical thinking.	PO 5. Explain and clarify mathematical thinking.	PO 6. Summarize mathematical information, explain reasoning, and draw conclusions.	PO 6. Summarize mathematical information, explain reasoning, and draw conclusions.	PO 6. Summarize mathematical information, explain reasoning, and draw conclusions.	PO 7. Isolate and organize mathematical information taken from symbols, diagrams, and graphs to make inferences, draw conclusions, and justify reasoning.
PO 6. Determine whether a solution is reasonable.	PO 6. Determine whether a solution is reasonable.	PO 6. Determine whether a solution is reasonable.	PO 7. Analyze and evaluate whether a solution is reasonable, is mathematically correct, and answers the question.	PO 7. Analyze and evaluate whether a solution is reasonable, is mathematically correct, and answers the question.	PO 7. Analyze and evaluate whether a solution is reasonable, is mathematically correct, and answers the question.	

The bulleted items within a performance objective indicate the specific content to be taught.

Mathematics Standard Articulated by Grade Level
Strand 5: Structure and Logic

Concept 2: Logic, Reasoning, Problem Solving, and Proof Evaluate situations, select problem-solving strategies, draw logical conclusions, develop and describe solutions, and recognize their applications.						
Kindergarten	**Grade 1**	**Grade 2**	**Grade 3**	**Grade 4**	**Grade 5**	**Grade 6**
			PO 8. Make and test conjectures based on data (or information) collected from explorations and experiments.	PO 8. Make and test conjectures based on data (or information) collected from explorations and experiments.	PO 8. Make and test conjectures based on data or information collected from explorations and experiments.	PO 8. Make and test conjectures based on information collected from explorations and experiments.
					PO 9. Identify simple valid arguments using *if…then* statements based on graphic organizers.	PO 9. Solve simple logic problems, including conditional statements, and justify solution methods and reasoning.
					PO 10. Construct *if… then* statements to generalize rules for computation, geometric properties and algebraic functions.	

The bulleted items within a performance objective indicate the specific content to be taught.

Mathematics Standard Articulated by Grade Level
Strand 1: Number and Operations

Concept 1: Number Sense Understand and apply numbers, ways of representing numbers, and the relationships among numbers and different number systems.			
Grade 7	**Grade 8**	**High School (Grades 9 and 10)**	**College Work Readiness (Grades 11 and 12)**
PO 1. Recognize and convert between expressions for positive and negative rational numbers, including fractions, decimals, percents, and ratios.		PO 1. Justify with examples the relation between the number system being used (natural numbers, whole numbers, integers, rational numbers and irrational numbers) and the question of whether or not an equation has a solution in that number system.	***PO 1. Solve problems and equations that require the number system to be extended from real to complex numbers.***
PO 2. Find or use factors, multiples, or prime factorization within a set of numbers.			
PO 3. Compare and order rational numbers using various models and representations.	PO 1. Compare and order real numbers including very large and small integers, and decimals and fractions close to zero.		
	PO 2. Classify real numbers as rational or irrational.	PO 2. Sort sets of numbers as finite or infinite, and justify the sort.	
	PO 3. Model the relationship between the subsets of the real number system.		
PO 4. Model and solve simple problems involving absolute value.	PO 4. Model and solve problems involving absolute value.	PO 3. Express that the distance between two numbers is the absolute value of their difference.	
			PO 2. Convert between radical and exponential forms of numerical expressions.

The bulleted items within a performance objective indicate the specific content to be taught.
The performance objectives highlighted in italics have been identified as core to an Algebra II course.

Mathematics Standard Articulated by Grade Level
Strand 1: Number and Operations

Concept 2: Numerical Operations Understand and apply numerical operations and their relationship to one another.			
Grade 7	**Grade 8**	**High School (Grades 9 and 10)**	**College Work Readiness (Grades 11 and 12)**
PO 1. Add, subtract, multiply, and divide integers.	PO 1. Solve problems with factors, multiples, divisibility or remainders, prime numbers, and composite numbers.	PO 1. Solve word problems involving absolute value, powers, roots, and scientific notation.	
PO 2. Solve problems with rational numbers and appropriate operations using exact answers or estimates.			
	PO 2. Describe the effect of multiplying and dividing a rational number by • a number less than zero, • a number between zero and one, • one, and • a number greater than one.	PO 2. Summarize the properties of and connections between real number operations; justify manipulations of expressions using the properties of real number operations.	
		PO 3. Calculate powers and roots of rational and irrational numbers.	***PO 1. Explore different forms of complex numbers; determine if the properties of the real number system extend to complex numbers and matrices.***
PO 3. Solve problems involving percentages, ratio and proportion, including tax, discount, tips, and part/whole relationships.	PO 3. Solve problems involving percent increase, percent decrease, and simple interest rates.		
PO 4. Represent and interpret numbers using scientific notation (positive exponents only).	PO 4. Convert standard notation to scientific notation and vice versa (include positive and negative exponents).	PO 4. Compute using scientific notation.	
PO 5. Simplify numerical expressions using the order of operations and appropriate mathematical properties.	PO 5. Simplify numerical expressions using the order of operations that include grouping symbols, square roots, cube roots, absolute values, and positive exponents.		***PO 2. Perform computations with complex numbers.***

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Mathematics Standard Articulated by Grade Level
Strand 1: Number and Operations

Concept 2: Numerical Operations Understand and apply numerical operations and their relationship to one another.			
Grade 7	**Grade 8**	**High School (Grades 9 and 10)**	**College Work Readiness (Grades 11 and 12)**
			PO 3. Describe the relationship between real and complex numbers including plotting complex numbers as points in a plane.
			PO 4. Define polar coordinates; relate polar coordinates to Cartesian coordinates.
			PO 5. Convert complex numbers to trigonometric form and then multiply the results.
			PO 6. Apply DeMoivre's Theorem to calculate products, powers, and roots of complex numbers.

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The performance objectives highlighted in italics have been identified as core to an Algebra II course.

Mathematics Standard Articulated by Grade Level
Strand 1: Number and Operations

Concept 3: Estimation Use estimation strategies reasonably and fluently while integrating content from each of the other strands.			
Grade 7	**Grade 8**	**High School (Grades 9 and 10)**	**College Work Readiness (Grades 11 and 12)**
PO 1. Estimate and apply benchmarks for rational numbers and common irrational numbers.		PO 1. Determine rational approximations of irrational numbers.	
PO 2. Make estimates appropriate to a given situation.	PO 1. Make estimates appropriate to a given situation.	PO 2. Use estimation to determine the reasonableness of a solution.	***PO 1. Recognize the limitations of estimations by assessing the amount of error resulting from estimation and determining whether the error is within acceptable tolerance limits.***
		PO 3. Determine when an estimate is more appropriate than an exact answer.	
PO 3. Estimate square roots of numbers less than 1000 by locating them between two consecutive whole numbers.	PO 2. Estimate the location of rational and common irrational numbers on a number line.	PO 4. Estimate the location of the rational or irrational numbers on a number line.	
PO 4. Estimate the measure of an object in one system of units given the measure of that object in another system and the approximate conversion factor.			

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Mathematics Standard Articulated by Grade Level
Strand 2: Data Analysis, Probability, and Discrete Mathematics

Concept 1: Data Analysis (Statistics) Understand and apply data collection, organization, and representation to analyze and sort data.			
Grade 7	**Grade 8**	**High School (Grades 9 and 10)**	**College Work Readiness (Grades 11 and 12)**
PO 1. Solve problems by selecting, constructing, and interpreting displays of data including multi-line graphs and scatterplots.	PO 1. Solve problems by selecting, constructing, interpreting, and calculating with displays of data, including box and whisker plots and scatterplots.	PO 1. Draw inferences about data sets from lists, tables, matrices, and plots.	PO 1. Solve problems by estimating and computing with one-variable and two-variable data.
		PO 2. Organize collected data into an appropriate graphical representation with or without technology.	
PO 2. Interpret trends in a data set, estimate values for missing data, and predict values for points beyond the range of the data set.		PO 3. Display data, including paired data, as lists, tables, matrices, and plots with or without technology; make predictions and observations about patterns or departures from patterns.	
	PO 2. Make inferences by comparing the same summary statistic for two or more data sets.	PO 4. Make inferences by comparing data sets using one or more summary statistics.	***PO 2. Compare data sets using graphs and summary statistics, including variance and standard deviation, with or without technology.***
PO 3. Identify outliers and determine their effect on mean, median, mode, and range.	PO 3. Describe how summary statistics relate to the shape of the distribution.	PO 5. Determine which measure of center is most appropriate in a given situation and explain why.	PO 3. Compute and explain summary statistics for distributions of data including measures of center and spread, including variance and standard deviation.
		PO 6. Evaluate the reasonableness of conclusions drawn from data analysis.	PO 4. Explain how sampling methods, bias, and the phrasing of questions asked during data collections impact the conclusions that can be drawn.
PO 4. Distinguish between a simple random and non-random sample.	PO 4. Determine whether information is represented effectively and appropriately given a graph or a set of data by identifying sources of bias and compare and contrast the effectiveness of different representations of data.	PO 7. Identify misrepresentations and distortions in displays of data and explain why they are misrepresentations or distortions.	PO 5. Identify misleading uses of data and explain why they are misleading.

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Mathematics Standard Articulated by Grade Level
Strand 2: Data Analysis, Probability, and Discrete Mathematics

Concept 1: Data Analysis (Statistics) Understand and apply data collection, organization, and representation to analyze and sort data.			
Grade 7	**Grade 8**	**High School (Grades 9 and 10)**	**College Work Readiness (Grades 11 and 12)**
	PO 5. Evaluate the design of an experiment.	PO 8. Design simple experiments or investigations and collect data to answer questions.	PO 6. Explain the differences between randomized experiments and observational studies and determine the appropriateness of using each in given situations.
			PO 7. Determine when arguments based on data mistake correlation for causation.
			PO 8. Draw a line of best fit for a scatterplot with or without technology, describe how the correlation coefficient relates to fit, and explain when it is appropriate to use the regression equation to make predictions.
			PO 9. Use matrices to organize and represent data.

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Mathematics Standard Articulated by Grade Level
Strand 2: Data Analysis, Probability, and Discrete Mathematics

Concept 2: Probability Understand and apply the basic concepts of probability.			
Grade 7	**Grade 8**	**High School (Grades 9 and 10)**	**College Work Readiness (Grades 11 and 12)**
PO 1. Determine conditional probabilities (experimental) in compound probability experiments.	PO 1. Determine theoretical and experimental conditional probabilities in compound probability experiments.	PO 1. Make predictions and solve problems based on theoretical probability models.	***PO 1. Apply probability concepts to calculate the probability of events and to make informed decisions in practical situations.***
	PO 2. Interpret probabilities within a given context and compare the outcome of an experiment to predictions made prior to performing the experiment.	PO 2. Determine the theoretical probability of events, estimate probabilities using experiments, and compare the two.	***PO 2. Use the principal characteristics of the normal distribution to estimate probabilities.***
			PO 3. Estimate probabilities and predict outcomes using one- and two-variable data.
PO 2. Experiment with two different events to determine whether the two events are dependent or independent of each other.	PO 3. Use all possible outcomes (sample space) to determine the probability of dependent and independent events.	PO 3. Use simulations to model situations involving independent and dependent events.	***PO 4. Determine the conditional probability of an event given that another event occurs, decide if two events are dependent or independent, and determine the probability of an event given the probability of the complementary event.***
PO 3. Compare the results of multiple repetitions of the same probability experiment to the theoretical probability.		PO 4. Explain and use the law of large numbers (that experimental results tend to approach theoretical probabilities after a large number of trials).	
PO 4. Compare probabilities to determine fairness in experimental situations.			
		PO 5. Use concepts and formulas of area to calculate geometric probabilities.	

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Mathematics Standard Articulated by Grade Level
Strand 2: Data Analysis, Probability, and Discrete Mathematics

Concept 3: Systematic Listing and Counting Understand and demonstrate the systematic listing and counting of possible outcomes.			
Grade 7	**Grade 8**	**High School (Grades 9 and 10)**	**College Work Readiness (Grades 11 and 12)**
PO 1. Analyze relationships among the tree diagrams where items repeat and do not repeat; make numerical connections to the multiplication principle of counting.	PO 1. Represent, analyze, and solve counting problems with or without ordering and repetitions.		
PO 2. Solve counting problems using Venn diagrams and represent the answer algebraically.	PO 2. Solve counting problems and represent counting principles algebraically including factorial notation.	PO 1. Apply the addition and multiplication principles of counting, representing these principles algebraically using factorial notation.	
		PO 2. Apply appropriate means of computing the number of possible arrangements of items using permutations where order matters, and combinations where order does not matter.	
		PO 3. Determine the number of possible outcomes of an event.	***PO 1. Use the binomial theorem and Pascal's Triangle to solve problems.***
			PO 2. Demonstrate the connections between the binomial coefficients, entries of Pascal's triangle, and combinations.

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Mathematics Standard Articulated by Grade Level
Strand 2: Data Analysis, Probability, and Discrete Mathematics

Concept 4: Vertex-Edge Graphs Understand and apply vertex-edge graphs.			
Grade 7	**Grade 8**	**High School (Grades 9 and 10)**	**College Work Readiness (Grades 11 and 12)**
			PO 1. Study the following topics related to vertex-edge graphs: Euler circuits, Hamilton circuits, the Travelling Salesperson Problem (TSP), minimum weight spanning trees, shortest paths, vertex coloring, and adjacency matrices.
PO 1. Use vertex-edge graphs and algorithmic thinking to represent and find solutions to practical problems related to Euler/Hamilton paths and circuits.		PO 1. Solve network problems using graphs and matrices.	PO 2. Understand, analyze, and apply vertex-edge graphs to model and solve problems related to paths, circuits, networks, and relationships among a finite number of elements, in real-world and abstract settings.
			PO 3. Devise, analyze, and apply algorithms for solving vertex-edge graph problems.
	PO 1. Use directed graphs to solve problems.		PO 4. Extend work with adjacency matrices for graphs, such as interpreting row sums and using the nth power of the adjacency matrix to count paths of length *n* in a graph.

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Mathematics Standard Articulated by Grade Level
Strand 3: Patterns, Algebra, and Functions

Concept 1: Patterns Identify patterns and apply pattern recognition to reason mathematically while integrating content from each of the other strands.			
Grade 7	**Grade 8**	**High School (Grades 9 and 10)**	**College Work Readiness (Grades 11 and 12)**
PO 1. Recognize, describe, create, and analyze numerical and geometric sequences using tables or graphs; make conjectures about these sequences.	PO 1. Recognize, describe, create, and analyze numerical and geometric sequences using tables, graphs, words, or symbols; make conjectures about these sequences.	PO 1. Recognize, describe, and analyze sequences using tables, graphs, words, or symbols; use sequences in modeling.	***PO 1. Analyze sequences and series and use them in modeling, including*** • ***explicit formulas for nth terms,*** • ***sums of finite arithmetic series, and*** • ***sums of finite geometric series.***
		PO 2. Determine a specific term of a sequence.	
		PO 3. Create sequences using explicit and recursive formulas involving both subscripts and function notation.	PO 2. Apply recursive formulas for arithmetic and geometric sequences to solve problems.
			PO 3. Distinguish between explicit and recursive formulas and convert between them, making good choices about when to use which.
			PO 4. Solve problems involving recursion.
			PO 5. Use and interpret sigma notation to represent summation.

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Mathematics Standard Articulated by Grade Level
Strand 3: Patterns, Algebra, and Functions

Concept 2: Functions and Relationships Describe and model functions and their relationships.			
Grade 7	**Grade 8**	**High School (Grades 9 and 10)**	**College Work Readiness (Grades 11 and 12)**
	PO 1. Sketch and interpret a graph that models a given context; describe a context that is modeled by a given graph.	PO 1. Sketch and interpret a graph that models a given context, make connections between the graph and the context, and solve maximum and minimum problems using the graph.	***PO 1. Express and solve problems that can be modeled using linear, quadratic, logarithmic, exponential, cubic, reciprocal, absolute value, and step and other piecewise-defined functions; interpret their solutions in terms of the context.***
	PO 2. Determine if a relationship represented by a graph or table is a function.	PO 2. Determine if a relationship represented by an equation, graph, table, description, or set of ordered pairs is a function.	
	PO 3. Write the rule for a simple function using algebraic notation.	PO 3. Use function notation; evaluate a function at a specified value in its domain.	***PO 2. Use function notation flexibly and evaluate a function at a value represented by an algebraic expression.***
	PO 4. Identify functions as linear or nonlinear and contrast distinguishing properties of functions using equations, graphs, or tables.		
PO 1. Use a table of values to graph an equation or proportional relationship; describe the graph's characteristics.	PO 5. Demonstrate that proportional relationships are linear using equations, graphs, or tables.	PO 4. Use equations, graphs, tables, descriptions, or sets of ordered pairs to express a relationship between two variables.	***PO 3. Graph absolute value, and step and other piecewise-defined functions identifying their key characteristics.***
			PO 4. Graph exponential functions identifying their key characteristics.
			PO 5. Sketch the graphs and determine the key characteristics of power functions in the form f(x) = ax^n , a≠ 0, for positive integral values of n.
			PO 6. Graph polynomial functions identifying their key characteristics.

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Mathematics Standard Articulated by Grade Level
Strand 3: Patterns, Algebra, and Functions

Concept 2: Functions and Relationships Describe and model functions and their relationships.			
Grade 7	**Grade 8**	**High School (Grades 9 and 10)**	**College Work Readiness (Grades 11 and 12)**
		PO 5. Recognize and solve problems that can be modeled using a system of two equations in two variables.	
		PO 6. Recognize and solve problems that can be modeled using a quadratic function.	
		PO 7. Determine domain and range of a function from an equation, graph, table, description, or set of ordered pairs.	***PO 7. Find domain, range, intercepts, zeros, asymptotes, and points of discontinuity of functions.***
			PO 8. Find the major and minor axes, intercepts and asymptotes of conic sections.
			PO 9. Find domain, range, intercepts, period, amplitude, and asymptotes of trigonometric functions.
			PO 10. Given a function • ***find the inverse of the function,*** • ***determine whether the inverse is a function,*** • ***explain why the graph of a function and its inverse are reflections of each other over the line y = x.***
			PO 11. Find approximate solutions for polynomial equations with or without graphing technology.

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Mathematics Standard Articulated by Grade Level
Strand 3: Patterns, Algebra, and Functions

Concept 2: Functions and Relationships Describe and model functions and their relationships.			
Grade 7	**Grade 8**	**High School (Grades 9 and 10)**	**College Work Readiness (Grades 11 and 12)**
			PO 12. Use theorems of polynomial behavior (including but not limited to the Fundamental Theorem of Algebra, Remainder Theorem, the Rational Root Theorem, Descartes Rule of Signs, the Conjugate Root Theorem) to find the zeros of a polynomial function.
			PO 13. Relate logarithms and exponential functions as inverses, prove basic properties of a logarithm using properties of its inverse, and apply those properties to solve problems.
			PO 14. Combine functions by composition, as well as by addition, subtraction, multiplication, and division including any necessary restrictions on the domain.
			PO 15. Determine if functions are even, odd, or neither both algebraically and graphically.
			PO 16. Identify the degree of a given polynomial function and write a polynomial function of a given degree.
			PO 17. Develop an informal notion of limits.

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Mathematics Standard Articulated by Grade Level
Strand 3: Patterns, Algebra, and Functions

Concept 3: Algebraic Representations Represent and analyze mathematical situations and structures using algebraic representations.			
Grade 7	**Grade 8**	**High School (Grades 9 and 10)**	**College Work Readiness (Grades 11 and 12)**
PO 1. Write a single variable algebraic expression or one-step equation given a contextual situation.	PO 1. Write or identify algebraic expressions, equations, or inequalities that represent a situation.	PO 1. Create and explain the need for equivalent forms of an equation or expression.	***PO 1. Rewrite and describe the need for equivalent forms of algebraic expressions.***
		PO 2. Solve formulas for specified variables.	
			PO 2. Apply the laws of exponents including rational and negative exponents to rewrite expressions in alternative forms.
PO 2. Evaluate an expression containing one or two variables by substituting numbers for the variables.	PO 2. Evaluate an expression containing variables by substituting rational numbers for the variables.		
PO 3. Solve multi-step equations using inverse properties with rational numbers.	PO 3. Analyze situations, simplify, and solve problems involving linear equations and inequalities using the properties of the real number system.		
PO 4. Translate between graphs and tables that represent a linear equation.	PO 4. Translate between different representations of linear equations using symbols, graphs, tables, or written descriptions.	PO 3. Write an equation given a table of values, two points on the line, the slope and a point on the line, or the graph of the line.	
		PO 4. Determine from two linear equations whether the lines are parallel, perpendicular, coincident, or intersecting but not perpendicular.	
PO 5. Create and solve two-step equations that can be solved using inverse operations with rational numbers.		PO 5. Solve linear equations and equations involving absolute value, with one variable.	
PO 6. Create and solve one-step inequalities with whole numbers.	PO 5. Graph an inequality on a number line.	PO 6. Solve linear inequalities in one variable.	

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Mathematics Standard Articulated by Grade Level
Strand 3: Patterns, Algebra, and Functions

Concept 3: Algebraic Representations Represent and analyze mathematical situations and structures using algebraic representations.			
Grade 7	**Grade 8**	**High School (Grades 9 and 10)**	**College Work Readiness (Grades 11 and 12)**
		PO 7. Solve systems of two linear equations in two variables.	***PO 3. Solve systems of three linear equations in three variables with or without technology.***
			PO 4. Use matrices to represent everyday problems that involve systems of linear equations.
		PO 8. Simplify and evaluate polynomials, rational expressions, expressions containing absolute value, and radicals.	***PO 5. Simplify radical expressions by performing operations on them.***
		PO 9. Multiply and divide monomial expressions with integer exponents.	
		PO 10. Add, subtract, and multiply polynomial and rational expressions.	***PO 6. Divide a polynomial by a lower degree polynomial.***
		PO 11. Solve square root equations involving only one radical.	
		PO 12. Factor quadratic polynomials in the form of $ax^2 + bx + c$ where a, b, and c are integers.	
		PO 13. Solve quadratic equations.	***PO 7. Find complex solutions for quadratic equations.***
		PO 14. Factor higher order polynomials.	***PO 8. Describe the relationships among the solutions of an equation, the zeros of a function, the x-intercepts of a graph, and the factors of a polynomial expression with and without technology.***
		PO 15. Solve problems using operations with matrices.	***PO 9. Use matrix operations and the inverse of a matrix to solve problems.***
			PO 10. Represent vectors as matrices.
			PO 11. Add, subtract, and compute the dot product of two-dimensional vectors; multiply a two-dimensional vector by a scalar.

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Mathematics Standard Articulated by Grade Level
Strand 3: Patterns, Algebra, and Functions

Concept 4: Analysis of Change Analyze how changing the values of one quantity corresponds to change in the values of another quantity.			
Grade 7	**Grade 8**	**High School (Grades 9 and 10)**	**College Work Readiness (Grades 11 and 12)**
PO 1. Use graphs and tables to model and analyze change.	PO 1. Interpret the relationship between a linear equation and its graph, identifying and computing slope and intercepts.	PO 1. Determine the slope and intercepts of the graph of a linear function, interpreting slope as a constant rate of change.	***PO 1. Analyze and describe how a change in an independent variable leads to a change in a dependent variable.***
	PO 2. Solve problems involving simple rates.	PO 2. Solve problems involving rate of change.	***PO 2. Identify patterns in a function's rate of change, including intervals of increase, decrease, and constancy; if possible, relate them to the function's verbal description or its graph.***
			PO 3. Analyze change in various contexts by modeling and solving word problems using functions and equations.
			PO 4. Compare relative magnitudes of functions and their rates of change.
		PO 3. Solve interest problems.	PO 5. Solve problems involving compound interest.
			PO 6. Demonstrate the relationship between • simple interest and linear growth and • compound interest and exponential growth.
			PO 7. Determine the total cost of purchasing consumer durables over time given different down payments, financing options, and fees.
			PO 8. Apply a variety of strategies to use tax tables and determine, calculate, and complete yearly federal income tax.

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Mathematics Standard Articulated by Grade Level
Strand 3: Patterns, Algebra, and Functions

Concept 4: Analysis of Change Analyze how changing the values of one quantity corresponds to change in the values of another quantity.			
Grade 7	**Grade 8**	**High School (Grades 9 and 10)**	**College Work Readiness (Grades 11 and 12)**
			PO 9. Develop a personal budget including debit, checking, and savings accounts by interpreting multiple personal budget examples.
			PO 10. Determine an effective retirement savings plan to meet personal financial goals including IRAs, ROTH accounts, and annuities.
			PO 11. Compare and contrast the role of insurance as a device to mitigate risk and calculate expenses of various options.

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Mathematics Standard Articulated by Grade Level
Strand 4: Geometry and Measurement

Concept 1: Geometric Properties Analyze the attributes and properties of 2- and 3- dimensional figures and develop mathematical arguments about their relationships.			
Grade 7	**Grade 8**	**High School (Grades 9 and 10)**	**College Work Readiness (Grades 11 and 12)**
PO 1. Recognize the relationship between central angles and intercepted arcs; identify arcs and chords of a circle.	PO 1. Identify the attributes of circles: radius, diameter, chords, tangents, secants, inscribed angles, central angles, intercepted arcs, circumference, and area.	PO 1. Use the basic properties of a circle (relationships between angles, radii, intercepted arcs, chords, tangents, and secants) to prove basic theorems and solve problems.	
	PO 2. Predict results of combining, subdividing, and changing shapes of plane figures and solids.	PO 2. Visualize solids and surfaces in 3-dimensional space when given 2-dimensional representations and create 2-dimensional representations for the surfaces of 3-dimensional objects.	
PO 2. Analyze and determine relationships between angles created by parallel lines cut by a transversal.			
		PO 3. Create and analyze inductive and deductive arguments concerning geometric ideas and relationships.	
PO 3. Draw and classify 3-dimensional figures with appropriate labels showing specified attributes of parallelism, congruence, perpendicularity, and symmetry.		PO 4. Apply properties, theorems, and constructions about parallel lines, perpendicular lines, and angles to prove theorems.	PO 1. Perform basic geometric constructions using a variety of methods, including • perpendicular bisector of a line segment, • bisector of an angle, and • perpendicular or parallel lines.
		PO 5. Explore Euclid's five postulates in the plane and their limitations.	PO 2. Explore geometries other than Euclidean geometry in which the parallel postulate is not true.
PO 4. Describe the relationship between the number of sides in a regular polygon and the sum of its interior angles.		PO 6. Solve problems using angle and side length relationships and attributes of polygons.	
		PO 7. Use the hierarchy of quadrilaterals in deductive reasoning.	

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Mathematics Standard Articulated by Grade Level
Strand 4: Geometry and Measurement

Concept 1: Geometric Properties Analyze the attributes and properties of 2- and 3- dimensional figures and develop mathematical arguments about their relationships.			
Grade 7	**Grade 8**	**High School (Grades 9 and 10)**	**College Work Readiness (Grades 11 and 12)**
PO 5. Identify corresponding parts of congruent figures.	PO 3. Use proportional reasoning to determine congruence and similarity of triangles.	PO 8. Prove similarity and congruence of triangles.	
		PO 9. Solve problems using the triangle inequality property.	
	PO 4. Use the Pythagorean Theorem to solve problems.	PO 10. Solve problems using right triangles, including special triangles.	
		PO 11. Solve problems using the sine, cosine, and tangent ratios of the acute angles of a right triangle.	PO 3. Apply the law of cosines and the law of sines to find missing sides and angles of triangles.
			PO 4. Use basic trigonometric identities including Pythagorean, reciprocal, half-angle and double-angle, and sum and difference formulas to solve equations and problems.

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Mathematics Standard Articulated by Grade Level
Strand 4: Geometry and Measurement

Concept 2: Transformation of Shapes Apply spatial reasoning to create transformations and use symmetry to analyze mathematical situations.			
Grade 7	**Grade 8**	**High School (Grades 9 and 10)**	**College Work Readiness (Grades 11 and 12)**
PO 1. Model the result of a double transformation (translations or reflections) of a 2-dimensional figure on a coordinate plane using all four quadrants.	PO 1. Model the result of rotations in multiples of 45 degrees of a 2-dimensional figure about the origin.	PO 1. Determine whether a transformation of a 2-dimensional figure on a coordinate plane represents a translation, reflection, rotation, or dilation and whether congruence is preserved.	
		PO 2. Determine the new coordinates of a point when a single transformation is performed on a 2-dimensional figure.	
	PO 2. Describe the transformations that create a given tessellation.	PO 3. Sketch and describe the properties of a 2-dimensional figure that is the result of two or more transformations.	
	PO 3. Identify lines of symmetry in plane figures or classify types of symmetries of 2-dimensional figures.		
		PO 4. Determine the effects of a single transformation on linear or area measurements of a 2-dimensional figure.	***PO 1. Describe how changing the parameters of a quadratic function affects the shape and position of its graph*** $f(x) = a(x-h)^2+k$***.***
			PO 2. Describe how changing the parameters of an exponential function affects the shape and position of its graph $f(x) = ab^x$***.***
			PO 3. Describe how changing the parameters of a trigonometric function affects the shape and position of its graph ($f(x) = A \sin B(x-C)+D$ or the other trigonometric functions).

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Mathematics Standard Articulated by Grade Level
Strand 4: Geometry and Measurement

Concept 3: Coordinate Geometry Specify and describe spatial relationships using rectangular and other coordinate systems while integrating content from each of the other strands.			
Grade 7	**Grade 8**	**High School (Grades 9 and 10)**	**College Work Readiness (Grades 11 and 12)**
	PO 1. Make and test a conjecture about how to find the midpoint between any two points in the coordinate plane.	PO 1. Determine how to find the midpoint between two points in the coordinate plane.	
	PO 2. Use the Pythagorean Theorem to find the distance between two points in the coordinate plane.	PO 2. Illustrate the connection between the distance formula and the Pythagorean Theorem.	
		PO 3. Determine the distance between two points in the coordinate plane.	
		PO 4. Verify characteristics of a given geometric figure using coordinate formulas for distance, midpoint, and slope to confirm parallelism, perpendicularity, and congruence.	
		PO 5. Graph a linear equation or linear inequality in two variables.	
		PO 6. Describe how changing the parameters of a linear function affect the shape and position of its graph.	
		PO 7. Determine the solution to a system of linear equations in two variables from the graphs of the equations.	***PO 1. Graph the solution set of a system of two or three linear inequalities and given an ordered pair determine whether it is a solution to the system.***
		PO 8. Graph a quadratic function and interpret *x*-intercepts as zeros.	***PO 2. Determine an equation of a circle given its center and radius; given an equation of a circle, find its center and radius.***
			PO 3. Graph equations of conic sections explaining the relationship between their algebraic form and key characteristics of the graph.

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Mathematics Standard Articulated by Grade Level
Strand 4: Geometry and Measurement

Concept 3: Coordinate Geometry Specify and describe spatial relationships using rectangular and other coordinate systems while integrating content from each of the other strands.			
Grade 7	**Grade 8**	**High School (Grades 9 and 10)**	**College Work Readiness (Grades 11 and 12)**
			PO 4. Graph all six trigonometric functions identifying their key characteristics.
			PO 5. Evaluate all six trigonometric functions at angles between (0 degrees and 360 degrees, 0 and 2π radians) using the unit circle in the coordinate plane.
			PO 6. Convert between rectangular and polar coordinates.
			PO 7. Graph equations given in polar coordinates.

The bulleted items within a performance objective indicate the specific content to be taught.
The performance objectives highlighted in italics have been identified as core to an Algebra II course.

Mathematics Standard Articulated by Grade Level
Strand 4: Geometry and Measurement

Concept 4: Measurement Understand and apply appropriate units of measure, measurement techniques, and formulas to determine measurements.			
Grade 7	**Grade 8**	**High School (Grades 9 and 10)**	**College Work Readiness (Grades 11 and 12)**
	PO 1. Solve problems involving conversions within the same measurement system.	PO 1. Use dimensional analysis to keep track of units of measure when converting.	PO 1. Explain, use, and convert between degree and radian measures for angles.
PO 1. Solve problems involving the circumference and area of a circle by calculating and estimating.		PO 2. Find the length of a circular arc; find the area of a sector of a circle.	
PO 2. Identify polygons having the same perimeter or area.		PO 3. Determine the effect that changing dimensions has on the perimeter, area, or volume of a figure.	
PO 3. Calculate the area and perimeter of composite 2-dimensional figures.			
PO 4. Determine actual lengths based on scale drawings or maps.	PO 2. Solve geometric problems using ratios and proportions.	PO 4. Solve problems involving similar figures using ratios and proportions.	
PO 5. Create a net to calculate the surface area of a given solid.	PO 3. Calculate the surface area and volume of rectangular prisms, right triangular prisms, and cylinders.	PO 5. Calculate the surface area and volume of 3-dimensional figures and solve for missing measures.	
PO 6. Identify the appropriate unit of measure to compute the volume of an object and justify reasoning.			
PO 7. Measure to the appropriate degree of accuracy and justify reasoning.			

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Mathematics Standard Articulated by Grade Level
Strand 5: Structure and Logic

Concept 1: Algorithms and Algorithmic Thinking Use reasoning to solve mathematical problems.			
Grade 7	**Grade 8**	**High School (Grades 9 and 10)**	**College Work Readiness (Grades 11 and 12)**
PO 1. Create an algorithm to determine the area of a given composite figure.	PO 1. Create an algorithm to solve problems involving indirect measurements, using proportional reasoning, dimensional analysis, and the concepts of density and rate.	PO 1. Select an algorithm that explains a particular mathematical process; determine the purpose of a simple mathematical algorithm.	
		PO 2. Analyze algorithms for validity and equivalence recognizing the purpose of the algorithm.	***PO 1. Use a variety of approaches (inductive and deductive reasoning, estimations, generalizations, formal and informal methods of proof) to analyze algorithms.***

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The performance objectives highlighted in italics have been identified as core to an Algebra II course.

Mathematics Standard Articulated by Grade Level
Strand 5: Structure and Logic

Concept 2: Logic, Reasoning, Problem Solving, and Proof Evaluate situations, select problem-solving strategies, draw logical conclusions, develop and describe solutions, and recognize their applications.			
Grade 7	**Grade 8**	**High School (Grades 9 and 10)**	**College Work Readiness (Grades 11 and 12)**
PO 1. Analyze a problem situation to determine the question(s) to be answered.	PO 1. Analyze a problem situation to determine the question(s) to be answered.	PO 1. Analyze a problem situation, determine the question(s) to be answered, organize given information, determine how to represent the problem, and identify implicit and explicit assumptions that have been made.	***PO 1. Analyze a problem situation, determine the question(s) to be answered, organize given information, determine how to represent the problem, and identify implicit and explicit assumptions that have been made.***
PO 2. Analyze and compare mathematical strategies for efficient problem solving; select and use one or more strategies to solve a problem.	PO 2. Analyze and compare mathematical strategies for efficient problem solving; select and use one or more strategies to solve a problem.	PO 2. Solve problems by formulating one or more strategies, applying the strategies, verifying the solution(s), and communicating the reasoning used to obtain the solution(s).	***PO 2. Solve problems by using theorems, formulating one or more strategies, applying the strategies, verifying the solution(s), and communicating the reasoning used to obtain the solution(s).***
PO 3. Identify relevant, missing, and extraneous information related to the solution to a problem.	PO 3. Identify relevant, missing, and extraneous information related to the solution to a problem.		
PO 4. Represent a problem situation using multiple representations, describe the process used to solve the problem, and verify the reasonableness of the solution.	PO 4. Represent a problem situation using multiple representations, describe the process used to solve the problem, and verify the reasonableness of the solution.	PO 3. Evaluate a solution for reasonableness and interpret the meaning of the solution in the context of the original problem.	***PO 3. Evaluate a solution for reasonableness and interpret the meaning of the solution in the context of the original problem.***
PO 5. Apply a previously used problem-solving strategy in a new context.	PO 5. Apply a previously used problem-solving strategy in a new context.	PO 4. Generalize a solution strategy for a single problem to a class of related problems; explain the role of generalizations in inductive and deductive reasoning.	***PO 4. Generalize a solution strategy for a single problem to a class of related problems and explain the role of generalizations in inductive and deductive reasoning.***
PO 6. Communicate the answer(s) to the question(s) in a problem using appropriate representations, including symbols and informal and formal mathematical language.	PO 6. Communicate the answer(s) to the question(s) in a problem using appropriate representations, including symbols and informal and formal mathematical language.	PO 5. Summarize and communicate mathematical ideas using formal and informal reasoning.	***PO 5. Summarize and communicate mathematical ideas using formal and informal reasoning.***

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The performance objectives highlighted in italics have been identified as core to an Algebra II course.

Mathematics Standard Articulated by Grade Level
Strand 5: Structure and Logic

Concept 2: Logic, Reasoning, Problem Solving, and Proof Evaluate situations, select problem-solving strategies, draw logical conclusions, develop and describe solutions, and recognize their applications.			
Grade 7	**Grade 8**	**High School (Grades 9 and 10)**	**College Work Readiness (Grades 11 and 12)**
PO 7. Isolate and organize mathematical information taken from symbols, diagrams, and graphs to make inferences, draw conclusions, and justify reasoning.	PO 7. Isolate and organize mathematical information taken from symbols, diagrams, and graphs to make inferences, draw conclusions, and justify reasoning.	PO 6. Synthesize mathematical information from multiple sources to draw a conclusion, make inferences based on mathematical information, evaluate the conclusions of others, analyze a mathematical argument, and recognize flaws or gaps in reasoning.	***PO 6. Synthesize mathematical information from multiple sources to draw a conclusion, make inferences based on mathematical information, evaluate the conclusions of others, analyze a mathematical argument, and recognize flaws or gaps in reasoning.***
	PO 8. Describe when to use proportional reasoning to solve a problem.	PO 7. Find structural similarities within different algebraic expressions and geometric figures.	***PO 7. Analyze and explain the general properties and behavior of functions or relations using algebraic and graphing techniques.***
PO 8. Make and test conjectures based on information collected from explorations and experiments.	PO 9. Make and test conjectures based on information collected from explorations and experiments.	PO 8. Use inductive reasoning to make conjectures, use deductive reasoning to analyze and prove a valid conjecture, and develop a counterexample to refute an invalid conjecture.	***PO 8. Use inductive and deductive reasoning to make, analyze, and validate or refute conjectures and/or proofs.***
			PO 9. Use mathematical models to represent and analyze personal and professional situations.
			PO 10. Differentiate, interpret, apply, and develop concepts in the context of personal and professional situations.
PO 9. Solve logic problems using multiple variables and multiple conditional statements using words, pictures, and charts.	PO 10. Solve logic problems involving multiple variables, conditional statements, conjectures, and negation using words, charts, and pictures.	PO 9. State the inverse, converse, and contrapositive of a given statement and state the relationship between the truth value of these statements and the original statement.	***PO 11. Determine under what conditions a given statement (algebraic, geometric) is true.***
	PO 11. Identify simple valid arguments using *if… then* statements.	PO 10. List related *if… then* statements in logical order.	
		PO 11. Draw a simple valid conclusion from a given *if…then* statement and a minor premise.	

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The performance objectives highlighted in italics have been identified as core to an Algebra II course.

Mathematics Standard Articulated by Grade Level
Strand 5: Structure and Logic

Concept 2: Logic, Reasoning, Problem Solving, and Proof Evaluate situations, select problem-solving strategies, draw logical conclusions, develop and describe solutions, and recognize their applications.			
Grade 7	**Grade 8**	**High School (Grades 9 and 10)**	**College Work Readiness (Grades 11 and 12)**
PO 10. Demonstrate and explain that the process of solving equations is a deductive proof.	PO 12. Make, validate, and justify conclusions and generalizations about linear relationships.	PO 12. Construct a simple formal deductive proof.	
PO 11. Use manipulatives and other modeling techniques to defend π (pi) as a ratio of circumference to diameter.	PO 13. Verify the Pythagorean Theorem using a valid argument.	PO 13. Identify and explain the roles played by definitions, postulates, propositions and theorems in the logical structure of mathematics, including Euclidean geometry.	

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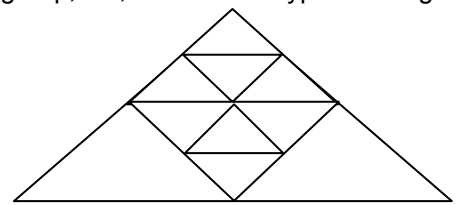
The performance objectives highlighted in italics have been identified as core to an Algebra II course.

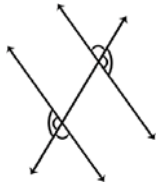
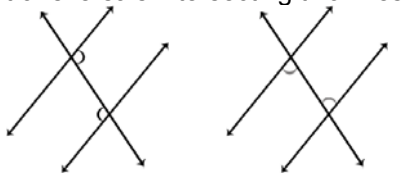
Mathematics Standard Articulated by Grade Level

GLOSSARY

The purpose of this glossary is to help the user better understand and implement the Mathematics Standard. It is not intended to be a study guide for the AIMS and is not a comprehensive list of all mathematics terms.

absolute value	a number's distance from zero on a number line (e.g., the absolute value of -4 is 4, the absolute value of 4 is 4; symbolically, $\lvert -4 \rvert = 4$ and $\lvert 4 \rvert = 4$)
absolute value function	a rule that defines a relationship between two sets of numbers that for each value of the independent variable set there is only one value in the dependent value set where $f(x) = \lvert x \rvert$, where $f(x) \geq 0$; for all values of *x*
accuracy (mathematical)	the extent to which a solution or measurement matches a standard or expected result
acute angle	an angle with measure between zero degrees and ninety degrees
addend	a number used in the mathematical operation of addition (e.g., 6 + 8 = 14, 6 and 8 are addends)
addition	a mathematical operation that combines two or more numbers to calculate a sum
addition principle of counting	a principle that allows for the efficient counting of the total number of ways a task can be accomplished when each part of the task consists of counting items from separate groups that do not overlap. For example, how many triangles are in the figure below? The task is to recognize there are three types of triangles (small, medium, and large) where each group does not overlap with another group; i.e., where each type of triangle appears as a member of one and only one group.  elementary school: If you want to count the total number of triangles in the figure above, count the number of small-sized triangles (8), count the number of medium-sized triangles (4), and count the number of large-sized triangles (1) and add them together (8 + 4 + 1 = 13). So there are a total of 13 triangles in the figure. If you have a task that can be accomplished through counting a collection of items among disjoint groups, and you count *m* items in the first group, *n* items in a second group, and *g* items in a third group (etc.,) then you can efficiently count the total number of items in the task by using the addition principle of counting. In this example, we would add *m* plus *n* plus *g* or (*m* + *n* + *g).* high school: let A_1 and A_2 be separate events that may occur at the same time with n_1 and n_2 possible outcomes for each event, respectively; then the total number of possible outcomes for the two events occurring are $n_1 + n_2$.

Mathematics Standard Articulated by Grade Level

adjacency matrix	the arrangement of rows and columns labeled by graph vertices, with a 1 or 0 in position (v_i, v_j) according to whether or not v_i and v_j are adjacent. For a simple graph with no self-loops, the adjacency matrix must have zeros on the diagonal while the adjacency matrix for an undirected graph is symmetrical
adjacent vertices	vertices joined by an edge or neighboring vertices in a vertex-edge graph
algebraic expression	a group of numbers, symbols, and variables that express a single or series of mathematical operations (e.g., $2x + 4 - 16y$)
algebraic form/notation	an algebraic description written in terms of numbers, symbols, and variables
algorithm	a set of step-by-step instructions for completing a task that can be generalized to other tasks, problems, or situations
alternate exterior angles	angles formed by one or more transversals intersecting two lines whose interiors are not between two lines and on different sides of the transversal (e.g., )
alternate interior angles	angles formed by one or more transversals intersecting two lines whose interiors are between the two lines and on different sides of the transversal (e.g., )
altitude of a geometric figure	a perpendicular segment from a base to a vertex or between bases
amplitude	a measure of one half the difference between the largest and smallest value of a function
analog clock	a device for the measurement of time that has numbers 1 to 12 around a face, with an hour, minute, and second hand that shows a continuous sweep of time
analyze	a process of dividing a composite into its parts for the purpose of examination
angle	a geometric figure consisting of the union of two rays that share a common endpoint (vertex)
angle bisector	a line, line segment, or ray that divides an angle into two congruent parts
angle measure	the measure (in degrees or radians) of the arc formed by two rays with a common endpoint (vertex)
annuity	a purchased investment contract between a person and an insurance company that defines payments to the insurer, in lump sum or in a series of payments, in exchange for benefits paid back to the insured at a designated date or series of dates

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appropriate	the reasonable use of an attribute, unit, or tool within the context of a problem (e.g., using a ruler to measure dimensions of a tissue box but not the dimensions of a building, using meters as the unit measure for the dimensions of a house but not the dimensions of a picture frame)
appropriate measure of accuracy	the degree of accuracy required for a mathematical task (e.g., approximating the lengths of lumber in framing carpentry requires less accuracy then the lengths of molding in finish carpentry)
approximation	a value or quantity that is close to, but not the same as, the desired value or quantity for a specified purpose
area	a two dimensional space measured by the number of non-overlapping unit squares or parts of unit squares that can fit into the space
arithmetic sequence	an ordered set of items in which the difference between each consecutive item is constant
arrangement	possible order of a set of events or items
array	a rectangular arrangement of objects or elements organized into rows and columns, or a set of objects or elements organized into a specific pattern
associative property	addition: changing the grouping of terms in a sum without changing the sum multiplication: changing the grouping of factors in a product without changing the product
asymptote	a line that a graph approaches
attribute of a figure	a property or common feature of a sets of objects or elements
attribute of a function or graph	a characteristic or distinct feature
average	the result of the sum of all the numbers in a data set divided by the number of elements in that data set
axis (axes: plural) (in two-dimensions)	one of two perpendicular number lines used to form a coordinate system
bar graph	a representation of the length of either vertical or horizontal bars used to enumerate and compare data
base	exponent: a term used to indicate a factor for repeated multiplication (e.g., in 4^7, 4 is the base) logarithm: the quantity *a* in the equation $x = \log_a y$
base of a polyhedron	the face of a geometric figure that identifies its type
base of a polygon	a side of a polygon that is perpendicular to its height
benchmark	a commonly known point of reference from which measurements may be made (e.g., four quarters make a whole)
benchmark fraction	a commonly known fraction that serves as a meaningful reference point for measurement comparison

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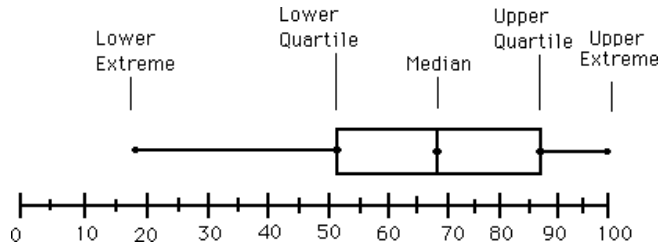
bias	sampling: a segment of data that is not representative of the original set of data statistical: an effect which deprives a statistical result of representativeness by systematically distorting it
binomial	an algebraic expression consisting of two terms (e.g., $x+3$, $4a-6$)
binomial theorem	a description of the coefficients of the expansion of the binomial $a+b$ raised to the *nth* power
bisect	to divide an object or term into two congruent parts
bisector	a point, segment, line, ray or plane which divides a segment, angle or figure into two parts of equal measure
box and whisker plot	a method for displaying the median, quartiles, and extremes of a data set 
brokerage fee	a fee in the form of a commission charged to the buyer by the brokerage firm for acting on behalf of the investor with the bond, commodities, or stock market
calculation	an action, process, or result of a mathematical computation
capacity	the amount of space in units or cubes that can fit into a solid (note: also referred to as volume)
Cartesian coordinate system	a plane containing points identified by their distance from the origin in ordered pairs along two perpendicular lines referred to as axes (note: also referred to as coordinate plane and rectangular coordinate plane)
causation	an agency or action that produces an effect
Celsius	a metric scale for the measurement of temperature based on the properties of water
central angle	an angle whose vertex is the center of a circle and whose sides (rays) are radii
chord of a circle	a segment whose endpoints are on a given circle
chromatic number	fewest number of colors needed to color a vertex-edge graph
circle	a set of points in a plane that are equidistant from a given point called the center
circle graph	a display of data as sections of a circle that represent all the data (note: formerly called pie graph or pie chart)
circuit	a path in a graph that starts and ends at the same vertex
circular arc	a fraction of the circumference of a circle
circumcenter	the point where the three perpendicular bisectors of the sides of a triangle meet
circumference	the total distance around a closed curve like a circle
coefficient	the number part of a term and variable combination (e.g., the coefficient for 7x is 7)
coincident	lines or shapes that have all points in common

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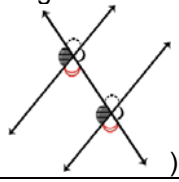
collinear	points that lie on the same line
coloring of a graph	assigning colors to the vertices of a vertex-edge graph so that adjacent vertices are assigned different colors
coloring of a picture or map	assigning colors to the regions of a picture or map so that regions that share a common border are assigned different colors
combination	The number of ways of choosing or selecting *k* unordered outcomes from *n* possibilities. $_nC_k = \binom{n}{k} = \frac{n!}{k!(n-k)!}$
common algorithm	a set of step by step instructions that are well known by most practitioners and are frequently used (e.g., borrowing, carrying)
common denominator	a number divisible by all of the denominators in a set of fractions
common factor	a whole number that divides without remainder into two or more non-zero numbers
common irrational numbers	a grouping of well known real numbers that cannot be expressed as a ratio of two integers (e.g., π , $\frac{\pi}{2}$, $\frac{\pi}{4}$, $\sqrt{3}$, $\sqrt{2}$, e)
common multiple	a whole number multiple of two or more given numbers (e.g., 48 is a common multiple of 2, 3, and 4)
commutative property	addition: the addition of terms in any order obtains the same sum (e.g., $a+b+c=d$, $a+c+b=d$) multiplication: the multiplication of terms in any order obtains the same product (e.g., $a*b*c=d$, $b*c*a=d$)
comparative language	words used to describe the differences in terms and objects (e.g., bigger, smaller, less than, more than, not equal to)
complementary angles	any two angles whose measures have a sum of ninety degrees
complementary events	two events whose probabilities of occurring sum to one; mutually exclusive events (e.g., when flipping a coin, getting a head and getting a tail are complementary events)
complete graph	a vertex-edge graph in which every vertex is adjacent to every other vertex
complex fraction	a fraction that has a fractional numerator, denominator, or both (e.g., $\frac{\frac{7}{11x^2}}{31y^5x^6s}$)
complex number	a number that can be written in the form $a+bi$ where *a* and *b* are real numbers and *i* is an imaginary number (e.g., 2+3*i* which is equivalent to $2+\sqrt{-3}$)

complex solution	a solution to a problem or equation that is not a real number
compose	to create by putting together
composite figure	a geometric figure that is composed of two or more simple polygons
composite number	a number that has factors other than one and itself
composition of function	a function comprised of more than one function arranged such that the output of one function becomes the input of the next function
compound interest	a percentage of an amount that accrues based on the product of the interest rate and the sum of the principal and any previously earned interest
compound probability	the likelihood that an event will occur based on whether another event has occurred
compound probability experiment	an organized process that examines the likelihood of two events occurring simultaneously, or the likelihood of one event occurring, instead of other possible outcomes, in conjunction with another event
computational estimation	the method of determining an approximate solution to a numerical problem
computational fluency	the efficient automatic recall of addition, subtraction, multiplication, and division facts; the efficient and automatic recall and use of standard algorithms for addition, subtraction, multiplication, and division
compute	to determine or calculate by mathematical means
conclusion	the *then* clause in an *if-then* conditional statement; a statement based on a reasonable judgment of two or more proposals
concrete materials	physical objects and manipulatives used for the purpose of instruction to represent mathematical situations
concrete situation	a condition derived from real-world examples and applications (note: also called contextual situation)
conditional probability	the likelihood that an event will occur based on an event that has already occurred
conditional statement	a statement with a hypothesis and conclusion in the form, *if* hypothesis, *then* conclusion (e.g., *if* a closed figure has exactly three sides, *then* the figure is a triangle)
cone	a three-dimensional figure generated by rotating a triangle about one of its legs to form a solid with one circular base
conflict	vertex-edge graphs can be used to model entities which are in conflict
congruent	having the same shape and exactly the same size
conic section	the intersection of a plane and two right conical surfaces that have the same vertex and whose angles are opposite rays (e.g., ellipse, parabola, hyperbola, circle)
conjecture	an unproven statement based on observations
Conjugate Root Theorem	if a+b*i* is a root for polynomial P, then a-b*i* is also a root for polynomial *P*

connected vertex-edge graph	a vertex-edge graph is connected if there is a path between all pairs of vertices (if a path does not exist between all pairs of vertices then the graph is disconnected)
consecutive	to follow in order one after the other
consecutive exterior angles	angles formed when one or more transversals intersect two parallel lines, that are not located between the two parallel lines, and are located on either side of a transversal
consecutive interior angles	angles formed when one or more transversals intersect two parallel lines, that are located between the two parallel lines, and are located on either side of a transversal
consecutive vertices	vertices that share a side of a polygon
constancy	the attribute of being unchanging, consistent, and regular
constant (of an expression)	a term with a degree of zero
constant (rate of change)	a fixed incremental increase or decrease over an interval
construct	arithmetic: the formation of a conclusion or the derivation of a result by joining or organizing forms geometry: to draw a geometric figure using appropriate tools to meet a given set of constraints
contextual situation	real-life scenarios or circumstances that illustrate mathematical problems (note: also called concrete situation)
contrapositive	a conditional statement that is the logical equivalent to the original statement exchanging the hypothesis with the conclusion and negating both of them
converse	a conditional statement that exchanges the hypothesis (*if)* and conclusion (*then)* components of an *if-then* statement
conversion factor	the ratio of two equal quantities that are measured in different units
convex polygon	a polygon with each interior angle measuring less than 180 degrees and whose diagonals lie inside the polygon
coordinate plane	a plane containing points identified by their distance from the origin in ordered pairs along two perpendicular lines referred to as axes (note: also referred to as Cartesian coordinate system and rectangular coordinate plane)
coordinates of a point	an ordered pair of real numbers that locate a point in a plane
correlation	the relationship between two or more data sets or variables
correlation coefficient	a value between 1 and -1 that determines if two lines have a linear relationship
corresponding angles	a pair of angles that occupy the same location at each intersection when two lines are intersected by one or more transversals (e.g., )

cosine	in a right triangle, the ratio of the length of the leg adjacent to a given acute angle to the length of the hypotenuse
counterexample	an example used to contradict or disprove a given statement
counting number	a number from the set of numbers consisting of 1, 2, 3, 4, 5, 6, … (note: also referred to as natural numbers)
counting problem	a type of problem that determines the number of arrangements, possibilities, or outcomes of events
cross-section	a plane section that intersects a solid
cube	exponents: the third power of a number geometry: a regular 3-dimensional figure having six congruent square faces
cube root	one of only three equal factors of a given number (e.g., the cube root of 27 is 3, 3 x 3 x 3 = 27
cubic function	a rule containing the cube of a variable (e.g., $f(x)=x^3$)
cycle graph	a vertex-edge graph where the vertices can be arranged in a circle so that each vertex is adjacent to the vertices that come before and after it
cylinder	a 3-dimensional figure composed of two congruent and parallel circular regions joined by a curved surface
data	quantitative and/or qualitative information within a context gathered through observation, questioning, and/or measurement
data set	a defined group of quantitative and/or qualitative information within a context gathered through observations, questioning, and/or measurement
De Moivre's theorem	a method to find the exponential value of an imaginary number; given any nonzero complex number z and any integer n, the n^{th} power of z, $r\text{CiS}(\theta)=r(\cos\theta+i\sin\theta)$ is $Z^M = (r(\cos\theta + i\sin\theta))^M = r^M((\cos n\theta + i\sin n\theta) = r^M \text{CiS}(n\theta)$
decimal point	a demarcation mark used in a base ten numbering system to designate values that are less than one
decompose	to break down into smaller units to simplify computation
deductive proof	a formal use of deductive reasoning using logical steps in the form of axioms, theorems, and given information
deductive reasoning	a series of logical steps in which a conclusion is drawn directly from a set of statements (premises) that are assumed to be true
degree	algebra: the degree of a term is the sum of the powers of each variable in the term geometry: a unit of measure based on dividing a circle into 360 equal parts, and used to measure angles, arcs and rotations temperature: the unit of measure for temperature
degree of a polynomial	the degree of the highest term of the polynomial
degree of a vertex	the number of edges that meet at a vertex in a vertex-edge graph
degree of accuracy	a standardized mathematic set of rules for rounding using significant figures that allows for the consistent handling of different scales of measurement
denominator	the bottom part of a fraction that indicates the number of equal parts into which the whole is divided (e.g., 4 in the fraction ¾)
density	the ratio of the amount of matter in an object compared to its volume; calculated as mass (m) per unit volume (v)
density property	a statement that says there is always a rational number between any two rational numbers

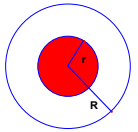
dependent events	two events such that the likelihood of the outcome of the second event is affected by the outcome of the first event
dependent variable	the output variable in a function which depends on the value of the input or independent variable
Descartes Rule of Signs	a mathematical method for the determination of both positive and negative zeros of a function; let P(x) be a polynomial with real coefficients: the number of positive zeros of P is either equal to the number of variations in sign of P(x) or less than this by an even number, and the number of negative real zeros of P is either equal to the number of variations in sign of P(−x) or less than this by an even number
descending	a sequential organizational method from biggest to smallest, greatest to least, latest to earliest
diagonal	a line segment joining two non-adjacent vertices of a polygon
diameter	a line segment that joins two points on a circle and passes through the center of the circle
difference	the result obtained using the operation of subtraction
digit	the ten symbols, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, used in a base ten numeration system
digital clock	a device for telling time that shifts between discrete states instead of continuous variation
dilation	a transformation that either enlarges or reduces a geometric figure proportionally using a scale
dimension	measurement: measure of distance in a specific direction (e.g., length, width, depth) space: the number of coordinates needed to specify a location in space
dimensional analysis	a use of proportional analysis as a problem-solving strategy for the conversion of measurement units
directed graph	a series of items linked by edges that are directed with an initial and terminal vertex (note: also referred to as digraph)
directrix	a fixed line perpendicular to the axis of symmetry and that lies the same distance from the vertex as the focus, but in the opposite direction
discount point	a fee assessed that is equal to 1% of the amount of a loan (e.g., one point on a $100,000 mortgage is equal to $1,000)
discrete	a condition in which the number of possibilities are separated from each other and are distinct
discrete mathematics	a contemporary branch of mathematics that is used in business, industry, and daily life; topics include combinatorics, iteration and recursion, and vertex-edge graphs
distance	the positive value for the length of the shortest line segment joining two points
distance formula	a general method or rule to measure the distance between two points that are identified by ordered pairs (e.g., $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$)
distortion in data display	the use of incorrect proportion, design variation in comparison of data sets, lack of context, or insignificant use of data in direct comparison with meaningful data
distribution of data	the values that a variable has across a spread of data
distributive property	a rule or method that states that every term inside grouping symbols may be multiplied by a term outside grouping symbols to yield an equivalent expression
dividend	the value to be divided in a division problem
divisibility	the ability to divide one whole number by another whole number without a remainder

Mathematics Standard Articulated by Grade Level

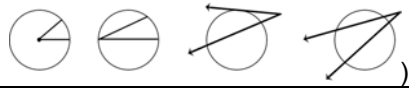
divisibility rules	a set of general rules that may be used to determine whether or not a number is evenly divisible by another number 2: if the number is even it is divisible by 2 3: if the sum of all of the digits is divisible by three, the number is divisible by 3 4: if the number formed by the last two digits is divisible by 4, the number is divisible by 4 5: if the last digit is a 0 or 5, the number is divisible by 5 6: if a number is divisible by both three and two, it is divisible by 6 7: if the difference of last digit doubled and the rest of the digits is divisible by seven, the number is divisible by 7 (e.g., 343: 34 – 6 = 28) 8: if the last three digits of a number are divisible by 8, the number is divisible by 8 9: if the sum of the digits is divisible by nine, the number is divisible by 9 10: if the last digit of the number is 0 it is divisible by 10
division	the opposite operation of multiplication that separates items or values into equal parts with or without a remainder
divisor	the value by which another quantity is divided in a division problem
documentary stamps	a state tax, in the form of stamps, that must be paid when ownership of a property passes from one owner to another
domain of a function	the set of values for the independent variable (input value) of a function
dot product	a mathematical operation that calculates a scalar product using two vectors and real numbers (e.g., for vectors $A= <X_a, Y_a>$ and $B=<X_b, Y_b>$ the dot product $A \cdot B=(x_a)(x_b) + (y_a)(y_b)$)
down payment	a portion of the full amount paid at the time of purchase or delivery
durables (consumer)	a product such as an automobile or appliance that has a life expectancy of at least three years
e.g.	the abbreviation for *for example*; precedes a non-exhaustive list of examples provided as options; other examples may be appropriate but not included (compare to i.e.)
edge (vertex-edge graph)	an edge or arc that connects two vertices in a vertex-edge graph or network
edge of a polyhedron	a line segment where two faces of a polyhedron intersect
efficiency (mathematical)	the ability to determine a method for solution quickly and with little effort
elapsed time	the measure of actual time between two distinct events
element	an item or term contained within a set of items or terms
ellipse	the set of all points in which the sum of the distances between focal points is a constant
ellipsis	a series of marks, "…", to indicate the continuance of a pattern or sequence
empty set	a set, signified by the symbol Ø, to indicate that the set contains no items or elements (note: also called the null set)
end behavior	a description of the performance of a function as it increases or decreases without boundaries
endpoint	a point that demarks the beginning and the end of a line segment, the initial point of a ray, or the end of an arc
equal	a term that indicates the same amount, measure, or quantity as another amount, measure, or quantity
equation	a mathematical statement divided by an equal symbol that states the two values or expressions have the same value

equilateral polygon	a polygon in which all sides are congruent
equivalent	two expressions or statements that always have the same truth value
estimate	an approximate and reasonable answer that is close to the exact answer without actually calculating the exact answer
Euclid's 1st Postulate	a line segment may be drawn joining any two points
Euclid's 2nd Postulate	any line segment can be extended indefinitely in a line
Euclid's 3rd Postulate	given a line segment, a circle can be drawn having the segment as a radius and one endpoint as a center
Euclid's 4th Postulate	all right angles are congruent
Euclid's 5th Postulate	only one straight line may be drawn between a given line and a point that is not on that line (note: also called the Parallel Postulate)
Euclidean geometry	the study of geometry based on definitions, undefined terms (point, line and plane), and the assumptions of Euclid
Euler circuit	a path in a vertex-edge graph that starts and ends at the same vertex and does not retrace any edges
Euler path	a path in a vertex-edge graph that travels every edge exactly once and the starting vertex differs from the ending vertex
evaluate	the use of one or more mathematical operations to calculate the value of an expression for a given input
even function	a function that meets the mathematical rule $f(x) = f(-x)$
even number	a natural number that is divisible by two without a remainder
even vertex	a vertex in a vertex-edge graph whose degree is even
event	outcomes during a probability activity
expanded notation	elementary: the display of digits to show the place value of each digit secondary: the display of an expression without parentheses
expected value	the average value distribution for a random variable
experimental (empirical) probability	a ratio formed by the comparison of the number of times an event occurs in an experiment to the number of times the experiment is completed
explicit	a statement that is expressed without ambiguity
explicit formula	an equation in which the dependent variable is written in terms of the independent variable (e.g., y=2x+3, $f(x)=x^5-7$, or I=Prt)
explicit sequence	a group of terms arranged in a predictable way (pattern) with a rule that is used to generate the n^{th} term of the pattern
exponent	a number placed to the right and above (superscript) a non-zero base that indicates the operation of repeated multiplication

exponential form	a mathematical representation of a term raised to a power or terms grouped and raised to a power (e.g., $5x^3$ or $(5x+7)^{2/5}$)
exponential function	an equation format written as $f(x) = a^x$ where the base, *a*, is a constant real number greater than zero but not equal to one.
exponential growth	the increase in a quantity over time represented by $y = a \cdot b^x$ where a > 0 and b > 1 (e.g., $y = 5(2)^x$; each time x is increased by 1, y increases by a factor of 2)
expression	a mathematical phrase containing one or more terms linked by operation symbols
extraneous	any data or information in a problem that is not necessary to determine a solution or to answer a question
extrapolation	to infer a value for an unknown variable in an interval using known values in a defined interval
extreme value	a maximum or minimum value of a function on a given interval
face of a polyhedron	each polygon that combines to construct a three-dimensional solid
fact family	a collection of related addition and subtraction facts, or multiplication and division facts, made from the same numbers (e.g., [7+2=9, 2+7=9, 9-7=2, 9-2=7} and {7X2=14, 2X7=14, 14÷7=2, 14÷2=7}
factor	noun: the value that can be divided into another value with no remainder verb: rewrite a number or polynomial as a product of numbers, simpler polynomials, or of polynomials and monomials
factorial	the product of all integers from a given number down to the number one
factorial notation	the format and symbol (!) used to represent a factorial
factoring	decomposing, through division, a complicated expression into the most simple expressions possible, that when multiplied yields the original expression
Fahrenheit	the U.S. customary or standard scale measure of temperature
fair sharing	the equal opportunity for the occurrence of all possible events or being equally divided
Fibonacci sequence	a recursive sequence in which every number is the sum of the two preceding numbers
financing	extending credit or purchasing on contract
finite set	a set of items or values that is limited to a countable number of elements
flexibility (mathematical)	a student's ability to recognize strategies necessary to complete a mathematical task, and a student's ability to apply learned strategies to alternative mathematical tasks
fluency	the efficient automatic recall of addition, subtraction, multiplication, and division facts; the efficient and automatic recall and use of standard algorithms for addition, subtraction, multiplication, and division
foci (of an ellipse)	two fixed points on an ellipse from which the sum of the distances of all other points on the ellipse is a constant
focus	a fixed point from which all other points are equidistant
formula	a general mathematical equation that relates two or more terms or values
Four Color Theorem	given any plane or spherical surface separated into regions, such as a political map of the states of a country, the regions may be colored using no more than four colors in such a way that no two adjacent regions receive the same color
fractal	a rough or fragmented geometric shape that can be subdivided into parts, each of which is (approximately) a reduced-size copy of the whole

fraction	a number written in the form of a ratio where the top number is referred to as the numerator and the bottom number is referred to as the denominator
fractional part	a part of a whole or a part of a group
frequency	the number of occurrences of an event within a specified interval
frequency table	a collection of data organized to display the number of events in a specified interval or multiple intervals
frieze pattern	a classification of patterns on two-dimensional surfaces that repeat in one direction
front-end estimation	using the leading, or left-most, digits to make an estimate quickly and easily (e.g., when asked to estimate the sum of 594, 32, and 221 an original estimate would be 5+0+2 hundreds or 700)
function (algebraic)	a rule that defines a relationship between two sets of numbers in that for each value of the independent variable set there is only one value in the dependent variable set
function notation	an equation in the form of $f(x) =$ to show the output value of a function, *f*, for an input value *x*
Fundamental Theorem of Algebra	an *nth* degree polynomial has *n* solution(s), real or complex
generalize	the ability to apply a solution method to many different problems and situations
geometric model	a representation of a geometric figure or concept
geometric pattern	a design representation of nonfigurative shapes including, lines, rectangles, and polygons
geometric probability	the likelihood of an event occurring based on geometric relationships such as area, surface area, or volume (e.g., if an arrow hits the target, the probability of hitting the red (shaded) bulls eye is $\frac{\pi r^2}{\pi R^2}$ 
geometric sequence	a finite or infinite progression of real numbers where each element is equal to the previous term multiplied by a constant referred to as the common ratio
geometric solid	a 3-dimensional shape bounded by surfaces (e.g., rectangular prism, pyramid, cylinder, cone, and sphere)
graph	a representation of an algebraic equation applied to a coordinate grid
graphic organizer	a visual tool designed to represent data in a format that improves understanding (e.g., Venn diagram, concept web, K-W-L chart)
greatest common factor	the largest natural number or monomial that divides into different natural numbers or terms without a remainder
grouping symbols	a variety of symbols of inclusion; parentheses, brackets, braces, or bars (i.e., ( ) , [ ] , {} , —, | |)
growing pattern	patterns that show an arithmetic or geometric change between pairs of elements in the pattern (e.g., numbers in decreasing order; buildings in decreasing size; or 3, 5, 8, 12, ……)
Hamilton circuit	a path in a vertex-edge graph that begins at a vertex, passes through every vertex exactly once, and returns to the original vertex

Hamilton path	a path in a vertex-edge graph that starts as some vertex in the graph and visits every other vertex of the graph exactly once.
height	a perpendicular segment from a base to a vertex or between bases (note: also called altitude)
hexagon	a polygon with six sides
higher order polynomial	an expression with a degree equal to or greater than two
histogram	a vertical bar graph with each bar representing a certain interval of data
horizontal	parallel to or in the plane of the horizon; in a coordinate grid, the x-axis is a horizontal line
hyperbola	a set of all points on a plane such that the difference between the distances from the plane to the foci is a constant, and is created by the intersection of the plane and the cone 
hypotenuse	the longest of the three sides of a right triangle; the side opposite the right angle in a right triangle
hypothesis	the *if* clause in an *if-then* conditional statement
i.e.	abbreviation for *that is*; precedes a specific list of items in which all of the items should be used (compare to e.g.)
identity element	a number when used in an operation with a given number leaves the given number unchanged
identity property of	addition: the rule that recognizes that a given number remains unchanged after the addition of a zero multiplication: the rule that recognizes that a given number remains unchanged after multiplication with the number one
image	a figure produced as the result of one or more transformations
imaginary numbers	the square root of a negative number expressed using *i* ($\sqrt{-1} = i$)
implicit	assumed or indirectly stated; inferred
implicit formula	an equation in which the dependent variable and independent variable are not separated by the equal sign, or in which the dependent variable is written in terms of the independent variable (e.g., 2x+y=3,= or $\frac{I}{P} = rt$)
improper fraction	a fraction in which the numerator is greater than the denominator
income tax	a monetary charge levied by an authority for public purposes that is based on monies made from employment, business, or capital gains
independent events	two events in which the outcome of the second event is not affected by the outcome of the first event
independent variable	the input value for a function
indirect measurement	a measurement determined without the direct application of measurement tools
indirect proof	a deductive reasoning strategy that uses contradiction or elimination to rule out all possible conclusions except the original statement which must be true

Individual retirement account (IRA)	an account that allows the holder to delay paying income tax and reduces the amount of taxes owed on the funds deposited
inductive reasoning	a reasoning process in which a conclusion is drawn from several observations
inequality	a statement relating two or more quantities or values that are not equal using words or symbols (≠, <, >, ≤, ≥)
inference	a conclusion drawn from given information, many times in the form of data
infinite set	the set in which the number of elements cannot be counted
input/output machine	a method used to build functions by applying a rule to an input value which generates an output value
inscribed angles	an angle with its vertex on the circle and with sides (rays) that are chords of the circle
integers	the set of real numbers consisting of the whole numbers and their opposites … -2, -1, 0, 1, 2 …
integral	general: an integer calculus: a function used for the calculation of the area under a curve
intercept	the point at which a line or curve crosses a given axis
intercepted arc	that part of a circle that lies between two segments, rays, or secants that intersect the circle (e.g., )
interpolation	a method for the estimation of the value of a function using the known values of a number above and below the unknown value
interquartile range	a measure of variability, that is resistant to outliers, determined by the difference between the first and third quartiles
interval	a set of numbers or values between, and in some cases including, two given values
inverse function	a function $f(y) = x$, denoted by $f^{-1}(x)$ such that the domain of the function $f(x)$ becomes the range of the inverse function $f^{-1}(x)$, and the range of $f(x)$ becomes the domain of $f^{-1}(x)$; the function will only have an inverse function if it is a one to one relation
inverse matrix	a rectangular array of values with columns and rows which when multiplied by the original array of values results in an array of values with a one for every diagonal element from the top left to the bottom right and a zero for all other elements in the array
inverse of a statement	a conditional statement obtained by negating both the hypothesis and the conclusion of a given conditional statement
inverse operation	a related but opposite process (i.e., multiplication is the inverse of division)
inverse relationship	additive: a number when added to a given number results in a sum of zero (note: also called identity property of addition) multiplicative: a number when multiplied to a given number results in a product of one (note: also referred to as the identity property of multiplication)
irrational numbers	a set of real numbers that cannot be expressed as a ratio of two integers (i.e., $\pi, \sqrt{2}$)

irregular polygon	a polygon whose interior angles are not equal and/or its sides are not equal in length
isosceles triangle	a triangle that has two or more congruent sides (note: equilateral triangles are a subset of isosceles triangles)
iteration	the repetition of a pattern or sequence
iterative pattern/ sequence	a pattern/sequence generated by using an initial value and repeatedly applying the same rule
justify	to prove or show to be true or valid using logic and/or evidence
kite	a quadrilateral with two distinct pairs of congruent adjacent sides and no congruent opposite sides
lateral face	a 2-dimensional surface that is not a base of a 3-dimensional figure
lateral surface	the sum of the lateral faces of a three-dimensional figure
Law of cosines	a law that allows for the calculation of the measurement of a side or angle of a triangle given other values for the triangle; for any ΔABC: $a^2 = b^2 + c^2 - 2bc\cos A$ $b^2 = a^2 + c^2 - 2ac\cos B$ $c^2 = a^2 + b^2 - 2ab\cos C$
Law of Large Numbers	the larger the sample the closer the experimental probability will approximate the theoretical probability
Law of sines	a description of the relationship between the angles of a triangle and the opposite sides of the same triangle; for any ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
least common multiple (LCM)	the smallest value for which two or more values are factors (e.g., the LCM of 3, 4, and 6 is 12, the LCM of $x^2 - 1$ and $x^2 - 3x - 4$ is $(x+1)(x-1)(x-4)$)
limit	the value, if one exists, that the dependent variable approaches as the independent variable approaches a given value
line	a straight set of points that extends infinitely in opposite directions (note: this is an undefined term in Euclidean geometry)
line graph	a representation used to show change over an interval, with the data points connected by line segments
line of best fit	a line drawn on a scatter plot to estimate the linear relationship among the data
line of reflection	the line that behaves as a mirror such that after a figure is reflected across the line all the points on the line are left unchanged by the reflection (transformation)
line of symmetry	a line that divides a figure into two congruent parts that are mirror images of each other
line plots	a sketch of data in which check marks, or other marks above a number line, shows the frequency of each value
line segment	two points or endpoints and all the points on the line between the endpoints
linear equation	an equation containing one or more terms in which the variable(s) is/are raised to the power of one but no higher
linear expression	a collection of numbers, symbols, operations, and two or fewer variables with a degree of one
linear function	a function that has a constant rate of change and can be modeled by a straight line

linear growth	a model for growth that adds a fixed amount to each time period
liter (L)	a metric unit of capacity that is equal to the volume of a cube that measures ten centimeters on a side
logarithm	a power to which a positive number base greater than one must be raised to generate a given number
logarithmic function	functions that involve logarithms and are the opposite of the exponential function
logic	a system of reasoning used to validate arguments
logic problem	a rational and varied systematic series of steps based on sound mathematical procedures in order to arrive at the solution
lowest common denominator (LCD)	the least common multiple of the denominators of every fraction in a given collection of fractions
magnitude	size or quantity
manipulatives	a wide variety of physical materials or objects that students use to foster the learning of abstract ideas in mathematics (note: also referred to as concrete materials)
mass	the amount of matter a body contains
mathematical argument	the justification of a particular solution, algorithm, or method using logic, evidence, and mathematically sound reasoning
mathematical fluency	the use of mathematical strategies with efficiency, accuracy, and flexibility
matrix	a rectangular array of numbers or letters arranged in rows and columns
maximum	the number with the greatest value in a set of numbers; the greatest vertical value in a graph
mean	a measure of center where the sum of a set of numbers is divided by the number of elements in the set (also referred to as the average)
meaningful context	the real world application of a mathematical concept
measurable attribute	a common feature of a set of objects or numbers that can be measured
measures of center	numbers that communicate the “center” or “middle” of a set of data (i.e., mean, median, and mode)
measures of spread	an indication of the dispersion or variation of data values including range, quartiles, interquartile range, standard deviation, and variance
median	a measure of center that identifies a value such that half the data is above the value and half the data is below the value when the data is listed in order
metric system of measurement	a measurement system based on the base-ten numeration system (e.g., meter, liter, gram)
midpoint	a point on a line segment halfway between the two endpoints
mid-spread	the difference between the upper and lower quartiles
minimum	the number with the smallest value in a set of numbers; the least vertical value in a graph
minuend	the number from which you are subtracting
mitigate	to cause to become less severe

mixed number	a number represented by a whole number next to a fraction, and is equal to the sum of the whole number and the fraction
mode	a measure of center that is the value or values that occur(s) most frequently in a given set of numbers
model (noun)	an object, drawing, graph, expression, or equation that represents a given context
Model (verb)	algebra and functions: choice of an equation or function to represent a given context geometry: use of physical objects or manipulatives to show a geometric situation
monomial/ monomial expression	an algebraic expression consisting of a single term that does not require any addition or subtraction (e.g., 5y)
multi-line graph	a representation consisting of two or more line graphs that correspond to discrete data sets
multiple of a number	a number into which a given number may be divided with no remainder
multiplication	the operation of repeated addition
multiplication principle of counting	a principle that allows for the efficient counting of the total number of ways a task is accomplished when some number of parts follows a first part of the task. For example, how many outfits can you make using three shirts, two pants, and four shoes? The task is to make an outfit with three parts (a shirt selection, a pants selection, and a shoes selection). elementary school: If you want to count the total number of ways a task can be completed that is accomplished through a series of parts, and you can select *m* ways to complete the first part, n ways to complete the second part, and *g* ways to complete the third part (etc.,) then you can efficiently count the total number of ways to accomplish the task by using the multiplication principle of counting. In this example, we would multiply *m* times n times *g* or (*m* • *n* • *g)* In the example above, we can count the total number of outfits by 3 x 2 x 4 or 24 outfits. high school: let A_1 and A_2 be events with n_1 and n_2 possible outcomes, respectively; then the total number of outcomes for the sequence of the two events is $n_1 \bullet n_2$high school: let A_1 and A_2 be events with n_1 and n_2 possible outcomes, respectively; then the total number of outcomes for the sequence of the two events is $n_1 \bullet n_2$
multi-variable equation	an equation with three or more variables that can be graphed in three or more dimensions
natural numbers	the set of real numbers consisting of 1, 2, 3, 4, 5, 6, … (note: also referred to as counting numbers)
necessary information	the values and statements required to find the solution to a problem
negation	statements meaning not or the opposite of; for any given statement p , its negation is the statement , ~ p (not p) whose truth value is the opposite of the truth value of p
negative number	a real number that is less than zero
neighboring vertices (of a vertex-edge graph)	vertices that share an edge (note: also referred to as adjacent vertices)

net of a polyhedron	a two-dimensional representation of the surface of a three-dimensional figure
network	A network or vertex-edge graph consists of a collection of vertices and edges where each edge connects two of the vertices
non-contextual problem	a problem given without an application/story
non-Euclidean geometry	a geometry that contains an axiom which is equivalent to the negation of the Euclidean parallel postulate *(e.g., R*iemannian geometry is a non-Euclidean geometry using the statement, "If l is any line and P is any point not on l, then there are no lines through P that are parallel to l" as its parallel postulate (also called elliptic geometry); and Hyperbolic geometry is a non-Euclidean geometry using the statement, "If l is any line and P is any point not on l, then there exists at least two lines through P that are parallel to l" as its parallel postulate
non-random sample	a sample selected using a biased method
non-routine problem	word problems that include a model of a real life situation, focus on higher levels of interpretation, are organized with no obvious solution, and that may require multiple problem solving strategies
non-standard shapes	geometric figures that are not in common usage but fulfill a given definition (e.g., )
non-standard units of measurement	measurement units that are not commonly accepted as standard but are applied uniformly when measuring (e.g., paperclips, pencils, a tennis shoe, and cubes)
normal curve	the symmetric statistical distribution of data evenly spread along a bell-shaped curve that reaches its maximum height at the mean
normal distribution	the spread of data that is symmetric in a given interval, has a median and mean that are equal, and can be fit with a normal curve
number line	a model that represents real numbers as points on a line with a uniform scale
numerator	the number of equal parts of a total number of parts in a fraction; it is found above the fraction bar (e.g., 4 in the fraction $\frac{4}{7}$)
numerical expression	any combination of constants, operators, and/or words that result in a number (note: also referred to as an arithmetic expression)
observable attribute	a common feature of a set of objects or numbers that is noticeable (can be observed)
observational study	a study attempting to infer the effects of an action in which the assignment of subjects to the group receiving the action and the group not receiving the action is outside the control of the observer
obtuse angle	an angle whose measure is greater than 90 degrees and less than 180 degrees
octagon	a polygon with eight sides
odd function	a function that meets the mathematical rule $f(-x) = -f(x)$
odd number	an integer that is <u>not</u> divisible by two
odd vertex	a vertex in a vertex-edge graph whose degree is odd

one-to-one correspondence	a relationship that pairs each element in a set with one element in another set
one-variable data	the data generated by one input cell used with a formula
operation	the process or execution of a specific rule on a set of numbers
order of operations	the sequence in which specific rules of mathematics are performed when evaluating an expression or equation
ordered pair	a pair of numbers used to locate and describe points in the coordinate plane in the form (x, y)
ordinal number	a whole number that names the position of an object in a set
ordinal position	numbers used to specify position in a sequence (e.g., first, second, third, fourth)
organized list	an orderly table of numeric or descriptive data used to solve a problem or an ordered plan to solve a problem
orientation	the arrangement of the points or objects, relative to one another, after a transformation; the direction traversed (clockwise or counterclockwise) when traveling around a geometric figure
origin	the intersection of the axes in a coordinate grid, often defined as (0, 0) in two-dimensions
origination fee	a charge levied by a lending institution for setting up a loan
outcome	a possible result for a probability experiment or simulation
outcome set	a set of all possible results for a probability experiment or simulation
outliers	numerical data that are significantly larger or smaller than the rest of the data in a set
parabola	the set of all points equidistant from the focus and the directrix
parallel lines	lines in the same plane that never intersect and are always equidistant
parallelism	a parallel relationship; the relation of opposition between things that will never intersect
parallelogram	a quadrilateral in which both pairs of opposite sides are parallel
parameter	algebraic/geometric: a quantity or constant whose value varies with the circumstances of its application statistical: a single number that describes some aspect of an entire population
Pascal's triangle	a triangular arrangement of numbers in which each row starts and ends with 1, and each other number is the sum of the two numbers above it 1 1 1 1 2 1 1 3 3 1 1 4 6 4 1 1 5 10 10 5 1
path (vertex-edge graph)	a connected sequence of edges that starts at a vertex and ends at a vertex
pattern	a set or sequence of figures or numbers that are repeated in a predictable manner
pentagon	a polygon with five sides

percent	a ratio that calculates the parts per hundred (e.g; 20% is 20 parts of 100)
perfect square	a whole number whose square root is a whole number
perimeter	the sum of all lengths of a polygon
period	the repeating interval of a periodic function
periodic function	a function that repeats itself at regular intervals
permutation	an ordered arrangement of a set of events or items
perpendicular lines	two lines that intersect to form right angles
perpendicularity	a perpendicular relationship; the relation of opposition between things at right angles
phase shift	the horizontal translation of a periodic graph
pi (π)	the ratio of the circumference of a circle to its diameter
pictograph	a representation that uses pictures or symbols to represent data
piece-wise defined function	a function that uses different rules for the number *x* depending on the element of the domain
place value	the value of a numeral based on the position of each digit in the number
plane	a 2-dimensional surface that extends infinitely in all directions (note: this is an undefined term in Euclidean geometry)
plane figure	a two-dimensional figure or shape formed by straight lines or a curve
point	a location in space that has no dimension (note: this is an undefined term in Euclidean geometry)
point of rotation	the point about which a figure is rotated or turned
points of discontinuity	a point where a function is not continuous, noted by an open circle on the graph of the function
polar coordinate system	a system in which a point on a coordinate plane is identified using its distance from the origin (r) and the positive angle (q) required to reach the point from 0° [e.g., (2, 40°)]
polygon	a closed two-dimensional figure made up of segments which intersect only at the segment endpoints
polyhedron	a closed three-dimensional figure or shape in which all the surfaces are polygons
polynomial/ polynomial expression	an expression containing more than one monomial connected by addition or subtraction
population	an entire set of objects that have something in common (e.g; animals with four legs, quadrilaterals, male students in Mr. R's class)
postulate	a mathematical statement that is accepted as true without proof
power	a quantity with a base and an exponent (e.g; x^5, where x is the base and 5 is the exponent)
precision	an indicator of how finely a measurement is made; it is related to the unit of measurement and the calibration of the tool
predictions	the use of base information to produce an approximation of change or result
pre-image	an object before it undergoes a transformation
premise	a statement that is given to be true

prime factor	all the factors of a quantity that are only divisible by the number one and itself (e.g; the prime factors of 42 are 7, 3, and 2; the prime factors of $6x^2y$ are 2, 3, x, x, and y)
prime factorization	the representation of the prime factors of a quantity
prime number	a number that has exactly two different factors, one and itself
prism	a three-dimensional figure made up of two parallel congruent faces and lateral faces that are parallelograms
probability	the measure of the likelihood of the occurrence of an event
product	the result obtained when two or more quantities are multiplied
proof	a sequence of logical arguments that prove a conjecture to be true
proper fraction	a fraction whose numerator is smaller than its denominator
properties of equality	rules for producing equivalent expressions (e.g., identity, transitive, reflexive, addition property of equality, to name a few)
properties of: operations, real number operations, real number system	mathematical principles that are always true (e.g., commutative, associative, distributive, identity, and inverse, to name a few)
proportion	the statement of equality between two ratios
proportional relationship	a relationship between two variables in which one is a constant (the constant of proportionality) times the other
proportionality	the concept of having equivalent ratios
proposition	a statement of truth that has yet to be proven
proximity	distance from an object
pyramid	a three-dimensional figure whose base is a polygon and whose lateral faces are triangles that share a common vertex
Pythagorean theorem	the statement that in a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse ($a^2+b^2= c^2$)
quadrant	one of the four sections into which the coordinate plane is divided by the *x*- and *y*-axes
quadratic equation	a polynomial equation containing one or more terms in which the variable is raised to the second power but no higher
quadratic formula	the formula used to find the roots (solutions) of a quadratic equation (i.e., $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$)
quadratic function	a function in the form: $f(x) = ax^2 + bx + c,\ a \neq 0$
quadrilateral	a polygon with four sides
quartiles	the four equally sized groups of data set
quotient	the answer to a division problem

radian/ radian measure	the measure of an angle formed by taking the radius of a circle and wrapping it along the circumference of the circle, the measure of an entire circle is 2π radians
radical	a symbol used to refer to the root of a number or term
radical expression	a mathematical expression containing one or more radicals
radical form	a term, expression, or equation that uses a radical instead of a fractional exponent(s) (e.g., writing the expression $\left(\sqrt[4]{2x}\right)^7$ instead of $(2x)^{\frac{7}{4}}$)
radius of a circle	the distance from the center of a circle to a point on the circle (plural: radii)
random sample	a sample in which each item or element of the population has an equal chance of being chosen as part of a sample of the population
randomized experiment	an experiment attempting to infer the effects of an action in which subjects are randomly assigned either to a group receiving the action or a group not receiving the action
range	the set of all possible output values for a function
range (of a data set)	the difference between the greatest and least value in a set of data
rate	a ratio comparing different types of measures (e.g., miles per gallon)
rate of change	the amount the function's output increases or decreases for each unit of change in the input
ratio	a comparison of two quantities by division that can be expressed as *a* to *b*, $\frac{a}{b}$, or *a*:*b*
rational expression	the quotient of two polynomials in the form $\frac{A}{B}$, where A and B are polynomials (e.g., $\frac{2x+1}{3x^2-9}$, $3x^2-9\neq 0$)
rational number	a number that can be expressed as a quotient of two integers
Rational Root Theorem	for a polynomial with integer coefficients, the only possible rational numbers that can be roots of the polynomial are ones of the form a/b, where a is a factor of the constant term and b is a factor of the leading coefficient
ray	a line segment that extends infinitely in one direction from one of its endpoints
real numbers	the set of rational and irrational numbers
re-allotment of square units	the application of the idea that what is changed in one place must be made up elsewhere in measurement problems involving square units
reasonable	within likely or sensible boundaries
reasonable estimations	approximations based on mathematical reasoning that are within the desired degree of accuracy (e.g., in the problem 35+43 a reasonable estimation would be 75 or 80)
reasoning (mathematical)	the justification of a particular solution, algorithm, or solution method using logical and mathematically sound arguments

reciprocal function	the function f(x) = 1/ax, where *a* is a constant and *a*≠0
reciprocals	two numbers whose product is equal to one (note also referred to as multiplicative inverses)
rectangle	a quadrilateral with two pairs of congruent parallel sides and four right angles
rectangular coordinate plane	a plane containing two perpendicular lines referred to as axes (note: also referred to as Cartesian coordinate system and coordinate system)
recursion	an inherently repetitive process by which the terms of a sequence can be computed from some or all of the preceding terms by an algorithmic procedure
recursive formula	a formula used to determine the next term in a sequence by using an algorithm with one or more of the preceding terms
recursive pattern	a pattern that uses the solution from previous steps to generate the solution to the next step (i.e., 2, 2, 4, 6, 10, 16…)
reflection	a transformation creating a mirror image of the original figure on the opposite side of the line of reflection
reflex angle	an angle that is greater than 180° and less than 360°
reflexive property	a property that states a quantity or figure is equal or congruent to itself
refute	to prove false by argument or evidence
regression equation	the equation for the line of best fit to a set of data points in the plane
regular polygon	a convex polygon which is equiangular and equilateral
relative magnitude	the value of numbers relative to a given value or number
Remainder Theorem	a theorem stating, "If f(x) is a polynomial, then the remainder obtained by dividing f(x) by x-r equals f(r)
repeating decimal	a decimal in which one or more digits repeats in a pattern without termination
repeating pattern	a sequence of figures or numbers that repeat in a predictable manner
representation	verb: the act of capturing a mathematical concept in some form noun: the form expressing a mathematical concept (e.g., equation, graph, model, written description, sketch, table, construction, manipulative)
revise	to change or modify based on evaluation
rhombus	a quadrilateral with four congruent sides (plural: rhombi)
right angle	an angle whose measure is 90°
right triangle	a triangle that contains a right angle
root	the solution (zeros) of a function
rotation	a transformation in which a figure is turned a given degree and direction around a point (the point of rotation)
ROTH account	an individual retirement arrangement that can be an account or annuity whose contributions are not tax deferred

round	to approximate the value of a number to a specified place value
sample	a part of the total population used in statistics to make predictions about the characteristics of the entire group
sample space	a list of all possible outcomes of an activity
scalar	a constant used in operations on matrices and vectors, distinguished from a vector or matrix in that it has size but not direction
scale	measuring: a tool or system used for the determination of weight graphing: a system of marks at fixed intervals
scale drawing	a reduced or enlarged drawing which is mathematically similar to the object which it represents
scale factor	the ratio between the lengths of corresponding sides of two similar figures
scalene triangle	a triangle with no congruent sides
scatterplot	a graph of the points representing a collection of data
scientific notation	a representation of a very large or very small number expressed as the product of a power of ten and a decimal number greater than or equal to one and less than ten
secant	a line that intersects a circle or some other curve at two points
sector of a circle	a region bounded by a central angle and its arc
sequence	a set of numbers in a defined order
series	the sum or difference of a sequence of numbers
shortest path	the path in a weighted vertex-edge graph from one vertex to another that has the least total weight
side	the segment joining two adjacent vertices in a figure
side length	the measure of the segment joining two adjacent vertices in a figure
sigma notation (Σ)	the Greek letter sigma used to indicate summation
similar figures	two or more figures that have the same shape and are related in size by a scale factor
simple interest	a fixed percent calculated on a principal amount without regard to accrued interest
simple polygon	a closed 2-dimensional figure that cannot be decomposed into closed figures with fewer sides without adding segments; a 2-dimensional figure whose sides do not cross through the interior of the figure
simplest form	fractional: a fraction that has no common factor for the numerator and denominator polynomial: an expression that has no common factors for all terms and no like terms radical: there are no perfect square factors contained in the radicand and there are no like terms
simplify	the act of writing a quantity in simplest form
simulation	an experiment to model a real-life situation for the purpose of examining a problem
sine	in a right triangle, the ratio of the length of the side opposite the given acute angle to the length of the hypotenuse
single event	one occurrence that can take place during a probability simulation that is not in conjunction with another occurrence
skip counting	the method of counting by equal intervals
slant height	pyramid: the altitude of a lateral face of a pyramid cone: the length of a line segment drawn on the lateral surface of a cone from its vertex to a point on its circular base

slope of a line	the measure of steepness of a line calculated as the change in *y* divided by the change in *x* (the rise over the run)
solid	a closed 3-dimensional figure
solution	the value or values for a variable that makes an equation or inequality true
solution methods	the strategy or set of strategies employed to solve a contextual or non-contextual problem
solution set	all the values that make an equation or inequality true
solve	to find a solution for a problem
space	the set of all points in three or more dimensions
spanning tree	a subgraph of a vertex-edge graph that is a tree and includes every vertex of the graph
sphere	a three-dimensional figure made up of all points in space equidistant from a given point called the center
spherical geometry	geometry applied to the surface of a sphere (note: this is a type of non-Euclidean geometry)
square	geometry: a parallelogram with four congruent sides and four right angles exponent: the result of multiplying a number by itself
square root	one of the two equal factors of a number
standard deviation	a statistical calculation of the dispersion of the data
standard notation	a number written with one digit for each place value in a base ten numeric system
statistics	the collection, organization, description, and analysis of quantitative data
stem-and-leaf plot	a display of data in which digits with larger place values (10's or greater) are "stems" and digits with smaller place values (1's) are "leaves" (e.g., 0| 0015888 1| 23445569 2| 3| 014 represents {0,0,1,5,8,8,8,12,13,14,14,15,15,16,19,30,31,34})
straight angle	an angle whose measure is 180°
subdivide	to decompose into smaller parts
subgraph	a portion of a vertex-edge graph that includes some of its vertices and some (or all) of its edges that connect those vertices
subscript	a number written to the right of and slightly below a term, usually used for indexing
subsets of a population	organizational groupings within a population
subsets of the real number system	organizational groupings of real numbers (e.g., rational numbers, irrational numbers, integers, whole numbers, natural numbers)
substitution property	the mathematical rule that allows equal values to replace each other
subtraction	a mathematical operation that calculates the difference between two numbers

subtrahend	the number being subtracted in a subtraction problem
sum	the result of addition
summary statistics	statistics used to summarize a set of observations, in order to communicate as much as possible as simply as possible; statisticians commonly try to describe the observations in three ways: • a measure of center, such as the arithmetic mean, median, mode, or interquartile mean; • a measure of statistical dispersion like standard deviation, variance, range, or interquartile range; and • a measure of the shape of the distribution like a normal curve
summation	the process of adding terms in a sequence for a given interval
supplementary angles	two angles whose measures have a sum of 180 degrees
surface area	a measure of the amount of area in a three-dimensional solid
symbol	shorthand marks that represent math concepts (e.g., $\leq, 4, \pm, \in, \angle, \pi$)
symmetric property	the mathematical rule that states for real numbers *a* and *b*, if *a* = *b*, then *b* = *a*
symmetry	a one-to-one correspondence in size, form, and arrangement of parts, related to a plane, line, or point
synthesize	the use of reasoning to combine sometimes diverse concepts or statements
system of equations	a set of two or more equations that must all be true for the same value(s) (note: also referred to as simultaneous equations)
systematic lists	an orderly listing of all possibilities for a given situation
table of values	a chart that organizes data (values) in rows and columns to illustrate facts and figures
tallies	a method of counting using marks usually in groups of five
tally chart	a method for recording occurrences of an event and for the development of frequency distribution tables
tangent	geometry: a line in the plane of a circle that intersects a circle at exactly one point trigonometry: in a right triangle, the ratio of the length of the leg opposite a given acute angle to the leg adjacent to the same angle
t-chart	a two column organizational tool used to display and record data, patterns, and functions/rules
term	a product or quotient of numerals, variables, or both; often separated by addition or subtraction operations in an expression
terminating decimal	a decimal that contains a finite number of digits
tessellation	one or more types of congruent figures that completely cover a plane without overlapping
theorem	a mathematical statement or proposition proven using previously accepted results
theoretical probability	the likelihood an event will occur under ideal circumstances divided by the total possible outcomes
tolerance	the allowable error in a given measurement
transformation	an operation that creates an image from a pre-image (e.g., translation, reflection, rotation, dilation, and glide-reflection)
transitive property	the rule stating that for real numbers *a, b,* and *c:* • *if a = b* and *b = c,* then *a = c;* • *if a > b and b > c then, a>c; and* • *if a < b and b < c, then a < c*

translate	the act of moving a figure in the coordinate plane preserving shape, size, and orientation
translation	a transformation that moves every point on a figure a given distance in a given direction
transversal	in a plane, a line that intersects two or more lines at different points
trapezoid	a quadrilateral that has exactly one pair of parallel sides
tree diagram	a representation used to find all the possible permutations for a set of items or the prime factorization of a number
trend	the general drift, tendency, or direction of data
triangle	a polygon with three sides
triangle inequality property	a property stating that, in a triangle, the sum of the lengths of two sides is greater than the length of the third side
trigonometric form	the form r(cos(θ) + I sin(θ)), where r is the magnitude of the complex number and θ is the angle it makes with the positive real axis
trigonometric functions	the functions sine, cosine, tangent, cotangent, secant and cosecant
trigonometric identities	equalities that are helpful for the simplification of complex trigonometric functions and that are true for every value of the variables (e.g., $\sin^2\theta + \cos^2\theta = 1$)
trigonometric ratios	the ratios of the lengths of pairs of sides in a right triangle (e.g., sine, cosine and tangent)
truth value	a value indicating whether a statement is true or false (note: typically written as sometimes true, always true, never true)
two-variable data	the data generated by two input cells used with one formula
unimodality	a function with one maximum during a defined interval
unit circle	the circle with a radius of one and center at the origin
unit fraction	a fraction with a numerator of one
unit rate	the ratio of a quantity to one unit of another quantity (e.g., unit price)
unnecessary information	information that does not assist with the solution to a problem
U.S. Customary system of measurement	a measuring system used most often in the United States (e.g., inches, pounds, gallons) (note: also called the standard system of measurement)
valid argument	an argument that is correctly inferred or deduced from a premise
variable	a symbol that represents a quantity

variance	population: a measure of variability given by the average of squared deviations if the data is taken from an entire population (i.e., $V_P = \frac{\sum(x_i - \overline{x})^2}{n}$) sample: a measure of variability given by the average of squared deviations if data is taken from a sample instead of an entire population(i.e., $V_S = \frac{\sum(x_i - \overline{x})^2}{n-1}$)
vector	a quantity that has magnitude (length) and direction
Venn diagram	a representation that uses circles to show relationships between two or more sets
verify	the process of demonstrating or proving that a response is correct
vertex	geometry: the point at which the rays of an angle, two sides of a polygon, or the edges of a polyhedron meet (plural: vertices) vertex-edge graph: vertices (singular "vertex') are elements or nodes of a graph or network that may or may not be joined by edges
vertex-edge graph	a graph or network that consists of a collection of vertices and edges where each edge connects two of the vertices
vertical	at right angles to the plane of the horizon or to a horizontal axis
vertical angles	the opposite angles formed when two lines intersect
volume	the measure of the capacity of a three-dimensional figure (measured in cubic units)
weight	a measure of the heaviness of, or the force of gravity on, an object
weight on an edge	value (or some number of objects) placed along an edge in a vertex-edge graph to represent some quantity such as distance, time, cost, or number of traffic lights
whole	the entire object, collection of objects, or quantity being considered
whole numbers	the set of numbers consisting of the natural numbers and zero
***x*-intercept**	the coordinate at which the graph of a line intersects the x-axis
***y*-intercept**	the coordinate at which the graph of a line intersects the y-axis
zero property	addition: the mathematical rule stating that the sum of a term and zero is equal to the original term subtraction: the mathematical rule stating that the difference of a term and zero is equal to the original term multiplication: the mathematical rule stating that the product of a term and zero is zero division: the mathematical rule stating that division of a term by zero is undefined
zeros (of a function)	the points at which the value of a function is zero (note: also called the roots of a function and the solutions for a function)